

PEDAGOGICAL STATISTICAL KNOWLEDGE OF A PROSPECTIVE TEACHER

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We present a case study of a prospective kindergarten and elementary teacher at the end of her teacher education program, when she had the opportunity to teach statistics in a classroom. With the aim of understanding her pedagogical statistical knowledge, we focus on her knowledge of students and knowledge of teaching. Results show that this prospective teacher reveals some knowledge of students and of teaching methods and strategies, but seems to have difficulty in translating that to practice.

INTRODUCTION

In a world dominated by numbers, teaching statistics is crucial to create citizens able to critique the information around them, knowing how to deal rationally with that information (Fabrizio, López, & Plencovich, 2007). To teach statistics, teachers need to know how to organize data, to interpret it, to build knowledge from it and to know when it is appropriate to do so (Batanero, Godino, & Roa, 2004). In addition to this statistical knowledge, pedagogical statistical knowledge is also required. As Batanero et al. (2004) state, teachers need to know how to help their students to develop correct intuitions in this field and how to deal with controversial ideas. Therefore, teacher education programs have an important role in developing both prospective teachers' statistical knowledge and pedagogical statistical knowledge. However, little research has been made to determine which knowledge teachers have and which is necessary to teach statistics (Chick & Pierce, 2008). In this paper we strive to understand the pedagogical statistical knowledge of a prospective kindergarten and elementary teacher (grades 1-4), focusing on her knowledge of students and knowledge of teaching.

CONCEPTUAL FRAMEWORK

Since Shulman's work (1986) that teachers' knowledge has been categorized as content knowledge and pedagogical content knowledge. Together with a sound statistical knowledge, teachers need pedagogical statistical knowledge to be able to effectively teach their students (Shulman, 1986). Sorto and White (2004) highlight knowledge of students and knowledge of teaching as key aspects of pedagogical statistical knowledge. In their view, knowledge of students involves the knowledge of conceptions and misconceptions, learning processes, students' work and classroom discourse. In addition, it includes the ability to interpret students' responses and to analyze their strategies and solutions in order to infer about their understanding. Batanero et al. (2004) also mention the ability to predict students' learning difficulties, mistakes, obstacles and problem-solving strategies. Studies that focus on teachers' knowledge of students indicate that they know little about students' statistical knowledge, their difficulties and ways to overcome them. Reporting on a study of 42 teachers, Watson, Callingham, and Donne

(2008) conclude that some of them make mistakes on items that call for a response to a particular misunderstanding of a student and cannot identify the next steps to take students to develop statistical understanding. Nicholson and Darnton (2003) also report that teachers are not familiar with students' common difficulties and misconceptions. González and Pinto (2008) conducted an investigation with four prospective teachers on statistical concepts of data organization and representation and concluded that they had no idea of students' difficulties on this topic. Consequently, they missed opportunities to help their students to confront these difficulties and achieve a deeper understanding of fundamental concepts.

Regarding the knowledge of teaching, Sorto and White (2004) report that teachers must have knowledge of teaching methods and strategies, explanations and didactic resources. For example, teachers should be able to recognize and take advantage of statistical everyday situations and use them to teach (Chick & Pierce, 2008). As for specific methods to teach this subject, emphasis is given in conducting experiments in the classroom, particularly in groups, to develop the understanding of certain concepts and linkages with other areas of the curriculum, taking into account students' interests and abilities (Ponte & Fonseca, 2001). Additionally, the GAISE report (Franklin et al., 2007) recommends that teachers: (i) emphasize the development of statistical literacy and statistical thinking and reasoning; (ii) use real and motivating data; (iii) emphasize the understanding of statistical concepts, as opposed to only teach theory and procedures; (iv) foster students' active role in the learning process; (v) make use of technology; and (vi) use assessment as a resource for improving teaching and learning. Concerning the teaching of graphs, Curcio's levels of graphs comprehension (1987) (read the data, read between the data and read beyond the data) are fundamental in teachers' knowledge to foster students' statistical literacy. Researching prospective teachers' difficulties regarding knowledge of teaching, Fernandes (2009) indicates that they have trouble in planning lessons and managing unforeseen situations and time. This author mentions that usually they adopt a more traditional approach (presentation of concepts, centered in the teacher, and practice of exercises). A study of Chick and Pierce (2008) shows that most of the prospective elementary teachers' lesson plans focus on correct graphs, rules of presentation and calculation of measures, but put little emphasis on understanding data sources and its implications. In another investigation, González and Pinto (2008) conclude that prospective teachers have no idea about the levels of understanding of statistical graphs.

METHODOLOGY

This study follows the interpretative paradigm and a qualitative approach, using a case study design with multiple data collection instruments: questionnaire, interviews, observations and document collection. Data analysis is descriptive and interpretative. The participant is a prospective kindergarten and elementary teacher (grades 1-4), with the fictitious name Dora, selected for being in the last years of a teacher education program with the possibility to teach statistics during her internship. In her 2nd year of studies (in 2010/11), she took the single course of the

teacher education program dedicated to statistics, given by the first author (who was also responsible for the data collection). During the course, prospective teachers worked statistical concepts through the exploration of situations in real contexts, with emphasis on representing and organizing data and statistical measures and their interpretation, and discussed how to pursue statistical investigations using different data collection instruments. In groups of up to three prospective teachers, they carried out a full statistical investigation, spending around 10 weeks. During her 3rd year of studies, Dora answered a questionnaire with content regarding statistics and didactics of statistics, being encouraged to present her line of thought. Her answers to this questionnaire (Qx) were afterwards discussed during interviews. During the 2nd semester of the 4th year (in 2012/13), she was enrolled in a supervised teaching course, having to teach a few days to a grade 2 class. Aside from an initial (EI) and a final interview (EF), Dora was asked to implement a statistical investigation with her students, which she divided in two different classes (in two consecutive days). Those classes were observed and video recorded and interviews were made before (EA) and after (ED) each class to discuss the plan and to reflect about what happened in class. Her reflections of the classes she taught (DB) were also collected.

RESULTS AND DISCUSSION

Dora is a very motivated and committed. She states that “up to the 9th grade [she] didn’t like [mathematics]” (EI), because “it was always the same. Always traditional. [The teacher] explained and then we did exercises” (EF). She mentions that this changed when, in high school, she decided to follow the area of Social and Human Sciences and the work “was more practical” (EI). Her enthusiasm for statistics is evidenced by the choice of this theme for her masters’ final report. She believes “it is very funny to work data organization and analysis with [children]” (EI) and says that she intends to “check which were the difficulties that they had, not only in the realization of graphs but also in their interpretation” (EF).

Knowledge of students

We discuss Dora’s ability to predict students’ difficulties, identify their mistakes and suggest ways to overcome these mistakes and her knowledge of students’ different strategies. With regard to anticipating students’ difficulties, in the questionnaire she had to build a frequency table and a plot for a data set and to formulate conclusions based on those representations. Later, she was asked to identify possible difficulties for 6th grade students in solving the previous tasks:

I believe that it may be at the level of the graph’s findings and in the construction of the frequency table, since the results are scattered, and thus there is the need to seek and count all equal results, which requires some concentration. (Q1.4)

We note that Dora identifies as difficult the data management (in the table construction or in the formulation of conclusions) since there are many and different data. When she had to justify her response, she stated “[students] could forget certain data” (EI). She considers that it will be difficult for students to organize and present

data, but doesn’t point out as difficult for them, for example, to draw conclusions that are more than a mere description of explicit data on representation.

During the statistical investigation that she conducted in the classroom, Dora took students to build a graph on which she wanted them to mark on the vertical axis their weights. Talking about students’ potential difficulties in doing this, she said “they will have a hard time making the graph, because of the scales” (EA). She did not point this difficulty in the previous situation since, possibly, she thought that this is no longer an obstacle for 6th graders. In fact, some of her 2nd graders showed problems in this direction, both in the choice of unit and in the correct marking of axis values:

Some students used the squares on the sheet of paper and placed the scale numbers inside, with no space for data that were in the range between two numbers (between 20 and 25, for example) (...) The bars of these two values became equal [20 and 22], since they couldn’t score data in the intervals of two numbers. (DB)

To address the difficulty of incorrect marking of the axis values, Dora decided that she should “convey some of that information so that groups could improve graphs” (DB). In fact, in the next day, she presented students “an example of another group” to discuss “what was the most perceptible and what was the most useful” (EF). In this situation, she decided to explain the correct way to make the graph with reference to one properly built. To lead students to overcome the obstacle of not choosing a constant unit on the vertical axis she said:

Dora: Now you have to keep in mind here in this side. That this here cannot be as it is in here. 1, 3 and then jump into the 7 and the 9; it must be a regular thing. How often can you make it? It’s like a sequence (...) If you are making 1 to 1, you cannot bring it up to... What is your maximum? (...)

Student: 29 (...)

Student: From 2 to 2.

Dora: From 2 to 2 or...

Student: 5 to 5. (Class)

Dora shows the need to explain how to properly mark the axis scale, inducing the unit 5. After reflecting on her lesson, she stated that students “should have had the opportunity to explore the situations mentioned [like using a scale with the unit 2], but due to lack of time this didn’t occur and I even had to induce them to this scale [of unit 5]” (DB). In this case, she takes control in overcoming students’ difficulties, using the strategy of explaining the correct procedure, which she explained by time constraints.

With regard to Dora’s ability to identify students’ errors and to suggest strategies to help students to overcome them, we showed her the following problem “There are 10 people in an elevator, 4 women and 6 men. The average weight of the women is 60 kg and the average weight of men is 80 kg. What is the average weight of the 10 people?” and a student’s solution showing a common misconception (figure 1).

$$\begin{array}{r} 80 \\ + 60 \\ \hline 140 \end{array} \quad \begin{array}{r} 140 \text{ L} \\ 00 \text{ 70} \\ 0 \end{array}$$

R: O peso médio das 10 pessoas é de 70 kg. R: The mean weight of the 10 people is 70kg

Figure 1: Student's response to a problem involving the concept of mean.

In the questionnaire, Dora wrote "I can't answer" (Q4.3.1), and this issue was brought up during one of the interviews:

- Dora: Ah, I can see now. He joined the mean of the 2 and divided by 2.
- Researcher: And why is it wrong?
- Dora: For me this is wrong because the 2 doesn't represent the total number of people.
- Researcher: So how should the student have done?
- Dora: (...) So, it should be divided by 10.
- Researcher: So, is it 14 kg?
- Dora: Well, I don't know. No, but he doesn't know the total number of... But is this really wrong? (...) For me it was easier to have the data of all men and women and divide by the number of people (...) There's not enough data (...) I don't know if this is possible to solve. (EI)

The difficulty of Dora in this matter is linked to the fact that she did not know what would be the correct answer and therefore an appropriate strategy (Q4.3.2), showing a strong link between content and pedagogical content knowledge. Some lack of understanding on the mean interferes with her pedagogical statistical knowledge, making it difficult to her to present strategies to help the student in this problem.

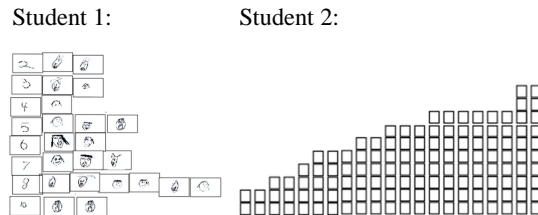


Figure 2: Representation of two students for the same data set.

Regarding the interpretation of students' strategies, Dora was shown two different representations, carried out by 2nd graders, depicting the number of teeth that each classmate had already lost (figure 2), and was asked to compare them. As a difference between them, Dora pointed that "the student 1 made the representation horizontally, while the student 2 made it vertically" and she saw no similarities (Q6.1). During the interview, she was asked again to explain these representations, and answered:

2 [students] lost 2 [teeth], 3 [students] lost 2 [teeth], 4 [students] lost 1 [tooth], [looking at student 1 representation] (...) I cannot remember how this [student 2] one here did (...) I don't know... because I know that this [representation of student 2] should show the same

as this one [representation of student 1]. [She turns the sheet in the opposite direction to realize if the representations have the same visual aspect] (...) But I think that to 2 students fell 10 [teeth] [looking to the last two columns of the student 2]. Oh, that's right! [looking to the representation of student 1], 2 students fell 10 [teeth]... That's it. (EI)

We observe that Dora began by explaining the representation of student 1, assuming that the numbers that this student wrote were the frequency and the cards with drawings in front of each number were the number of teeth. When she tried to explain the representation of student 2, Dora seemed to realize that her understanding of the first student's representation was not quite right, since they should represent the same information. When looking at the last two columns of the representation of student 2, she concluded that each column in this case is the number of teeth lost by each student and that only thinking in this way the two representations conveyed the same information. However, faced with the question of which representation she thought to be more complex, she chose the representation of student 2, since initially "I didn't understand it. I needed to ask him (...) Now I get it, because I have this [representation of student 1] right here" (EI). Although she only properly understood the representation of student 1 after analyzing the representation of student 2, she continued to find that the representation of student 2 was more complex. This demonstrates that she associates the complexity of students' representations to the difficulty or not in interpreting them. Another reason may be due to the link of the cards' drawings of student 1 to a more childish representation.

Knowledge of teaching

We also discuss the different teaching methods and strategies Dora used during the classes that she taught or that she indicated to follow. In the statistical investigation that she conducted with her 2nd graders, it was discussed students' gain of weight. Dora considered that students can and should assume control at different phases of this process: "early in the investigation, I [should] give them room to talk. They were a little anxious and nervous. And they were very restless, then that also eventually led me to be controlling" (EFD). Nevertheless, in her classroom, she did not allow for much independence from students during the investigation, namely in posing a problem, making a plan and choosing representations (we show this in detail in Santos & Ponte, 2014).

In what concerns the type of work with students, Dora adopted several times group work. She showed knowledge of students in the constitution of groups since her decisions were very productive and efficient, according to her supervisor teacher. When elaborating groups she took "in consideration the existence of a leader, to help to organize work and that requires some elements to follow him/her" (DB). Yet, management of group work was one of her main concerns:

For example, tomorrow I think will be very hard to manage all groups and help them (...) If they were used to this, to this type of work, maybe it wouldn't be so difficult. Now, I know this will be the first time they will do work like this (EI).

Dora was very worried about the independent work of the different groups, considering that students were not used to work on open tasks. She was also concerned with her role in the classroom, since she did not have experienced this type of work as student: “I never did something like this. I never did a statistical investigation with them (...) It seems different (...) To teach other things, they were things I learned in school and that they did with me. Now this, no” (EI). However, in class, her activity was impressive, being aware and following each group work. She also shows that to teach statistics it is important for students to be main actors of their own learning: “Firstly I think it should always come from them, which means, do statistics with them (...) Which is, for them to collect data, organize it in tables or as they want, then construct graphs and, then, from there, take conclusions” (EF).

Concerning teaching strategies to work the analysis of graphs, Dora did some research on her own. Besides showing the perspective of the importance of elaborating hypothesis during the construction of graphs, during the analysis of a pictogram in the classroom, she encouraged her students to examine the representation, which may allow them to confirm or contradict their initial hypotheses:

I started with the question “What can be said about this graph?”, since it’s a very open question and allows a diversity of answers, that sometimes we wouldn’t have access if we were only to ask the mode, minimum, what data have the same values, etc. (...) To conclude the analysis of the pictogram I placed another question to the group: “How many more boys like snakes than cats?” (DB).

First she places an open question to see what the focus of different students is, and then she poses a question that requires a reading between the data, to go beyond the literal reading of information. During the analysis of the graph built during the weights investigation that she led in classroom, she also took into account the literature she read about questioning, such as the connections with other mathematical concepts, like the case of the estimation, and the attempt to break some stereotypes regarding weight, speaking of students’ height: “This work was very interesting, especially when we held it in a large group, because I think I made an interesting exploration of the graph, touching in key aspects such as comparison between weights and heights and estimation” (DB).

In the statistical investigation that she conducted with students, Dora considered that the new topics for them were the formal aspects of graphing (title, scales, axis, labels) Initially, she had no clear idea about how to introduce this: “I also don’t know very well” (EA). Dora predicted some difficulties regarding the scales, where she controlled the discussion, taking students to adopt the unit of measure she considered appropriate. She justified this type of classroom decisions at certain moments by saying that it is important to take students to get to the most useful methods: “Besides being the students to suggest the data organization, the intern’s role will always be to take the group to think if that is the most advantage record method” (DB). To discuss the importance of the graph’s title, she prepared two graphs “one with nothing and

the other with that kind of information for them to verify which one gives the most complete information” (EA) and took students to compare the information on them:

- Dora: I have here these two graphs (...) Which one do you think it is best represented? Most perceptive?
- Student: This is wrong, isn’t it? Because I’m not the heaviest.
- Dora: No, this isn’t the weight (...) Do you know, from this graph, what is it about?
- Student: No.
- Dora: And from this? Can you see?
- Student: It’s the height.
- Dora: Is this the height? What does it say here?
- Student: Number of times that goes to the movies each month. You go more times to the movies.
- Dora: From this you already know what it is (...) Put a title.
- Student: Title?
- Dora: Yes, it should have a title so that we know what it is about. (Class)

In this classroom excerpt, we observe that to introduce the graph conventions, Dora used the explanation and the supply of properly constructed examples for students to consider the advantages of presenting this information. In another situation proposed in an interview, she had to plan the introduction of the concept of mode to a group of students. Dora said that she “could grab these materials [math cubes] and began by asking which is the most prevalent color in this set. Then from there introduce the mode concept” (EF). Thus, to present this concept she thought of a task with manipulative materials. It should also be noted that she did not consider explaining the concept first and then applying it. On the contrary, she seemed to initiate the application of the concept and only then introduced the proper denomination. When asked to indicate the difference between concepts being introduced through a task or through its emergence during a statistical investigation, she said:

It’s different. The fact that sometimes it arises spontaneously turns out to work better, on the experience I have (...) If I took a task already made to work the mode... It’s different. I can’t explain (...) Also because the context there would be different. We would already be talking about other statistical concepts; here we would only be facing the mode concept (...) I don’t know if it would be better, or if it would be worse. It would be different (...) And depends on the children’s experience (...) It’s just different (EF).

Thus, Dora sees many advantages in using statistical investigations in the classroom, but indicates that it is not necessary to perform an investigation whenever she wants to introduce a new concept. However, we observe that she gives great importance to teaching statistics through investigations, not only to learn concepts, but also for other reasons:

Through these investigations, I think they end up gaining much more not only regarding their mathematical learning, but also from other areas, especially at the personal and social level. Since they are in groups, they have to listen, know how to be heard, respect others' ideas and that area ends up to be developed and worked (EF).

CONCLUSION

About Dora's pedagogical statistical knowledge, we conclude that she has some knowledge of students, predicting correctly some difficulties in problematic situations (elaborate the graph scale, organize and present data), contradicting Nicholson and Darnton's findings (2003). To help students overcome their obstacles, she decides to explain the suitable procedures or, sometimes, compares it to a more appropriate solution. However, she demonstrates that she cannot discern the complexity of different data representations. These difficulties may be associated with shallow knowledge or no understanding of certain statistical concepts and procedures, and are thus connected with some flaws in statistical knowledge.

This prospective teacher demonstrates to know and make use of teaching strategies and methods recommended by several authors (Curcio, 1987; Franklin et al., 2007; Ponte & Fonseca, 2001): group work, open ended tasks, statistical investigations, use of real data, different levels of graph analysis. This was, probably, due to her special interest in the theme and her study of the literature. In class, sometimes, she ends up adopting a more expository teaching, explaining correct procedures (construction accurately of a graph with reference to the title and scale), which she mainly justifies by time constraints, as pointed by Fernandes (2009). Some difficulties that she points are mainly due to lack of experience as a student with this kind of work.

To Groth (2007), it is essential that the learning of statistics and its didactics, by prospective teachers, do not occur separately, but in the same course. Thus, the ideal would be that opportunities, during the teacher education program, for open ended tasks and for teaching methods to develop an understanding of statistical concepts could help to increase prospective teachers' experience as students, but also serve as discussion moments of what is required to conduct this kind of work in their future with students. During the internship, this work has to be continued, with moments of preparation and reflection about classes. Possibly, this is a job that must be continued during the induction period and even afterwards, given the complexity involved in the construction of knowledge of all topics and its didactics.

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