

# The development of prospective elementary school teachers' knowledge in geometric measurement

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There is a large agreement, based on empiric evidence, around the idea that mathematics teaching should base on a deep knowledge of the content (not just the amount of knowledge but also in the way it is organized) and in a pedagogic knowledge of the content, understood as the way teacher transforms the mathematical knowledge so that this can be taught and understood by the students (e. g. Ball & Bass, 2003; Ball, Lubienski & Mewborn, 2001; Hill, Rowan & Ball, 2005; Jaworski & Gellert, 2003; Kahan, Cooper & Bethea, 2003; Ponte & Chapman, 2006; Shulman, 1986). This fact has great implications in teachers' education, especially in elementary school teachers, for which mathematics is a general requirement rather than a specialized subject (Adler et al., 2005). In spite of the expectation that these professionals will be able to provide quality instruction to their pupils, most of them have poor prerequisites and low expectations towards mathematics.

Throughout their path as elementary and secondary school students, prospective teachers developed personal beliefs on teaching, perspectives concerning the role played by teachers and students and concerning the nature of the Mathematics, and through these he/she will interpret his/her own training as teachers to be (Oliveira, 2004). The way teachers see mathematics constitutes a central aspect in the definition of his/her own mental model on teaching and learning and it will profoundly shape the way they'll teach their students. (Perkkila, 2003; Hannula, Kaasila, Laine & Pehkonen, 2005). This "vision of the Mathematics" is an aspect of knowledge but is also in the one of beliefs and conceptions. Pehkonen & Pietila (2003), making reference to Furinghetti & Pehkonen (2002), consider two types of knowledge: objective knowledge - Formal, official, public - in the case of mathematics, it's the knowledge created and compiled by the mathematicians along the centuries; and subjective knowledge - individual, personal - it's the knowledge based on the experiences and acquired understanding of each individual. It is in this last category of knowledge that we include beliefs, a central construct in Mathematics Education research (e.g. Abelson, 1979; Cobb, 1986; Ernest, 1994; Hannula, Kaasila, Laine & Pehkonen, 2005; Leder, Pehkonen, & Toner (eds.), 2002; McLeod, 1992; Pehkonen & Pietila, 2003; Perkkila, 2003; Pajeras, 1992; Schoenfeld, 1983).

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Geometry and Measurement is an important chapter in the elementary school mathematics curriculum. The concept of measurement plays an essential role in the construction of the intricate web of geometric conceptualizations, reasoning and applications (Battista, 2007). In a wider level than other mathematics topics, measurement relates very closely to others, such as fractions and decimals, geometry, and also with natural and social sciences. The majority of references on teaching and learning measurement focus on children understandings of magnitudes such as angle amplitude, length, area and volume (e. g. Battista, 2004 and 2007; Lehrer, 2003; Nunes et al., 1993; Clements & Brighth (eds.), 2003) in cognitivistic perspectives. Stephan et al., 2003, present research focusing on social contexts. Few researchers studied the understandings on this subject by students who will be teachers, in the perspective of mathematical knowledge for teaching (Baturó & Nason, 1996).

With this study I intend to characterize the knowledge in geometric measurement held by prospective elementary school teachers. I'm also interested in understanding how these students, still in an early stage of their training, develop, embody and integrate the content knowledge acquired in this course, how they combine it in their beliefs systems related to mathematics and its teaching and the role played by their scholastic and cultural backgrounds. The approach to this research will be a qualitative one, as I intent to consider some of the students as subjects for case studies. Cognition, culture and social aspects are expected to be taken in consideration in the theoretical framework for the analysis of the collected data. I didn't formulate yet my research questions in an explicit manner. My research will take place during the fall semester of the next academic year, in a class of prospective elementary school teachers, as I teach a course in Geometry and Measurement (it was meant to be taken place in the present semester but it was not possible). The work developed in this course will be based on problem solving and investigative and exploratory activities. I expect to have several sources of data: Individual interviews to the participants in the study, where some tasks will be proposed to the students, classroom activities for groups of students, recorded in audio and video, and also posts and discussions in a learning management system platform, which work as an e-learning complement of the attendance of classes. The analysis process will elapse as the data go being collected, so that that analysis can supply new emerging categories of the data, in order to provide conditions for some theorization on the theme.

Presently, I'm doing my background readings (most of them among the references in the end of this document), while trying to clarify in my head the categories of analysis of the collected data, according to the chosen perspectives. I'm focusing my readings on Teachers Subject Matter Knowledge,

Beliefs and Knowledge, Mathematical aspects of Geometric Measuring (e. g. looking at Chevallard's work) and methodological aspects, mainly, the choice of theoretical approaches. My present dilemma is this choice. Can I conciliate socio-cultural with cognitive perspectives? I'm looking at embodied and distributed cognition as a mean in establishing links with socio-cultural and socio-constructivist perspectives. After this present step, I hope to be able to formulate more precise research questions. In the mathematics of geometric measuring it's important to establish what future elementary school teacher need to know on the subject, what mathematics is involved in measuring and what useful connexions (in the perspective of the students as prospective teachers) can be made with other subjects.

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