



# A Framework and a Research Design Proposal to Identify Preservice Teachers' Integration Performance of Science and Mathematics

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**Abstract** Integration is an important, albeit difficult, goal of science and mathematics education. This paper presents a framework and a research design proposal to identify preservice teachers' (PSTs) integration performance of science and mathematics (IPoSM). It also presents the classification of PSTs' level of integration in each phase of a problem-based learning (PBL) activity, within the proposed framework, and evidence of their work. The framework is organized in three levels of integration, based on current research, and detailed with specific integration indicators about an exemplary PBL activity, named *STEM Bees*. This PBL activity was structured as a research problem divided into two integrated problems. A qualitative methodology was used, focused on the application of the framework to the work of PSTs in that PBL activity. The final participants were 13 PSTs, organized in four working groups. In general, results show that PSTs' integration performance was higher with the second problem than with the first problem. However, full integration with meaningful connections between science and mathematics ideas was achieved by almost all PST at the conclusion of the research problem. The IPoSM framework indicators proved to be useful for identifying the level of integration of science and mathematics in PSTs' work in each phase of the PBL activity. Keeping the IPoSM framework in mind, teacher educators could elaborate more effective problem-based integrated approaches of science and mathematics in initial teacher education.

**Résumé** Bien que difficile à réaliser, l'intégration constitue un objectif important de l'enseignement des sciences et des mathématiques. Cet article présente un cadre ainsi qu'un projet conceptuel de recherche destinés à déterminer le rendement des enseignants en formation initiale (EFI) dans leur processus d'intégration des sciences et des mathématiques (RdISM). On y présente aussi la catégorisation

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du niveau d'intégration des EFI dans chacun des stades d'une activité d'apprentissage par résolution de problèmes (ARP), à l'intérieur du cadre proposé ainsi que des preuves de leur travail. En accord avec la recherche en cours, on a organisé le cadre en trois niveaux d'intégration comprenant des indicateurs d'intégration précis applicables à une activité type d'ARP nommée «Abeilles STEM». Cette activité d'ARP était conçue de façon à agir comme problématique de recherche que l'on a divisée en deux enjeux intégrés. On a utilisé une méthode qualitative ciblée sur la mise en application du cadre au travail réalisé par les EFI lors de cette activité d'ARP. La liste finale des participants comprenait 13 EFI placés dans quatre groupes de travail. En général, les résultats montrent un rendement en intégration de la part des EFI plus élevé dans le deuxième enjeu que dans le premier. Toutefois, l'intégration complète comprenant l'établissement de liens étroits entre les concepts scientifiques et mathématiques a été atteinte par presque tous les EFI au terme de la problématique de recherche. Les indicateurs du cadre de RdISM ont démontré leur utilité pour déterminer le niveau d'intégration en sciences et en mathématiques dans le travail des EFI pour chacun des stades de l'activité d'ARP. En tenant compte du cadre de RdISM, les formateurs d'enseignants pourraient concevoir des approches d'intégration fondées sur la résolution de problèmes plus efficaces en sciences et en mathématiques dans la formation initiale des enseignants.

**Keywords** Framework · Initial teacher education · Integration of science and mathematics · Preservice teachers · Problem-based learning

## Introduction

Mathematics and science integration has been researched for a long time (McHugh et al., 2018; Ríordáin et al., 2016) and many attempts have been made to integrate it in the curriculum. For example, one of the Portuguese curriculum principles highlights the importance of “valuing collaborative and interdisciplinary work on the planning, implementation and evaluation of teaching and learning” (Decree-Law 55/2018, p. 2931).

Although there are numerous studies on the methods, design, and classroom practices of science and mathematics integrated education, An (2017) states that there are no effective and replicable methods in preservice teacher (PST) education to develop instruction that highlights the connections between science and mathematics. There is a lack of deep questioning and methodologies to analyse the integration of science and mathematics. In fact, the comprehensive literature review done by Czerniak and Johnson (2014) shows that more research on integration is needed, including about methods for preparing teachers to educate using an interdisciplinary approach. Further studies have recommended empirical research on methods for advancing teacher education programmes with a focus on interdisciplinary pedagogy (e.g. An, 2017; An & Tillman, 2018). Integration is the key factor in interdisciplinary education (Greef et al., 2017), so more research is required to assist teachers with interdisciplinary practices and to foster the development and application of substantial curriculum frameworks and resources about integration (English, 2016).

In fact, the literature lacks a framework for identifying PSTs' integration performance of science and mathematics (IPoSM). Therefore, this study aims to present that framework and a research design proposal to develop such framework and to identify PSTs' performance in science and mathematics integration. With that focus, PSTs' work on one problem-based learning (PBL) science and mathematics integrated activity is analysed using the proposed framework and research design. PBL has received increased attention as a pedagogical approach for integrating subjects in initial teacher education contexts (Navy & Kaya, 2020; Navy et al., 2021) and is used in the presented study to foster science and mathematics integration.

Aiming to advance the knowledge and understanding of science and mathematics education, the research question that guided the study was: To what extent did PSTs integrate science and mathematics in their work on a PBL activity?

## Theoretical Framework

### Integration of Science and Mathematics in Initial Teacher Education

In their overview of curriculum integration, Wall and Leckie (2017) state that the meaning of curriculum integration differs from source to source, and that it is integrated by teachers and schools in different ways. This section will provide a discussion of the meaning of integration, with a focus on initial teacher education.

Beane (1997) argues that curriculum integration is a curriculum design theory whose aim is to enhance personal and social integration through significant problems and issues, identified both by educators and students, without concern for subject-area lines. The approach used by Beane (1997) is similar to that found in Honey et al. (2014), who define integration as “working in the context of complex phenomena or situations on tasks that require students to use knowledge and skills from multiple disciplines” (p. 52).

Some researchers state that the idea of integration is often used with the same meaning as interdisciplinary approach (e.g. Lederman & Lederman, 2013). On the contrary, Beane (1997) advocates that integration transcends subject-area and disciplinary identifications, while the interdisciplinary approach retains subject-area and disciplinary distinctions around a unifying theme.

Beane (1997) also distinguishes different types of integration, among which is integration of knowledge. This type of integration happens when the content-area concepts are integrated through projects and other activities, focusing on issues, aiming to lead to meaningful learning, the integration of experiences and the application of different skills. There are some similarities between the previous conceptions and the work of Greef et al. (2017), who add that integration can also be defined as the synthesis of two or more disciplinary visions, obtained from different perspectives, with new knowledge. This idea aligns with Huntley’s (1998) understanding of integration, which places integration at one end of a continuum starting at intradisciplinary, moving on to interdisciplinary to then achieve integration. About the continuum idea of integration, Beane (1997) suggests that it cannot be enough to distinguish what real integration is from what it is not. For this author, real integration only happens when it is not possible to retain the identity of separate subjects in new designs.

Integration is an important, albeit a difficult, goal of science and mathematics education (Koirala & Bowman, 2003). The literature suggests that effective and integrated science and mathematics education could help improve preservice teacher education (Czerniak & Johnson, 2014), and provide opportunities for less fragmented knowledge and more relevant and stimulating learning experiences (Frykholm & Glasson, 2005). It is known that PSTs value integration (Berlin & White, 2010; Koirala & Bowman, 2003). However, to achieve successful integration in their practices, PSTs must experience integrated methods in their teacher education courses (Koirala & Bowman, 2003; McHugh et al. 2018). This is the reason why You (2017) alerted to one of the most important problems of integrated education: “If nobody teaches students how to integrate the different disciplines, students are prevented from discovering and creating links between relevant subject matter.” (p. 73). There is also evidence that courses focused on science and mathematics integration methods can refine PSTs’ understanding of constructivism (Jones et al., 2005), and develop their interdisciplinary science inquiry (Chowdary et al., 2014) and interdisciplinary pedagogical content knowledge (e.g. An, 2017; An & Tillman, 2018).

However, the teachers' implementation of science and mathematics integration faces many challenges, also identified in teacher education (An, 2017). Some of those challenges are deeply rooted academic traditions in teacher training (Samson, 2014). Institutions are frequently divided into distinct departments or academic fields (Mason, 1996), which results in science teachers still being trained and certified to teach separate disciplines (Lederman & Lederman, 2013; You, 2017), a common problem in mathematics teacher education. Other barriers to integration are PSTs' perceptions of the difficulties and challenges of integration (Berlin & White, 2010), and confidence about teaching mathematics and science (Baxter et al. 2014; Weinberg & McMeeking, 2017). Tension and difficulties that can arise in the integration of certain topics (Koirala & Bowman, 2003), difficulties regarding content knowledge (Frykholm & Glasson, 2005; You, 2017) and weak content knowledge integration (You, 2017) are relevant aspects that can condition integration. The fact that PSTs usually lack integration experiences during initial teacher education (Brown & Bogiages, 2019; Koirala & Bowman, 2003) can also be a barrier for their confidence in interdisciplinary approach. You (2017) highlights that if teachers have insufficient content knowledge in the specific disciplines, they will also lack the ability to integrate the concepts. Research also identifies that teachers experience difficulties in planning and teaching integrated lessons (Czerniack & Johnson, 2014; Widjaja et al., 2019) and lack the time for planning collaboratively (Widjaja et al., 2019). In fact, creating integrated tasks is a big challenge and requires more time than specific tasks for just one subject, as stated by Brown and Bogiages (2019).

Evaluating the integration of science and mathematics is another challenge. There is a variety of frameworks for supporting and analysing integration of science and mathematics. However, they are mainly focused on curriculum (e.g. Huntley, 1998; Kiray, 2012; McHugh et al., 2018), or teaching and learning strategies for integration (e.g. Aguirre-Muñoz et al., 2021), or a combination of both (e.g. Davidson et al., 1995). For example, Huntley's (1998) framework represents a science and mathematics continuum during instructional practice with different levels of integration: mathematics for the sake of mathematics, mathematics with science, mathematics and science, science with mathematics and science for the sake of science. A framework for identifying PSTs' performance on different levels of science and mathematics integration (e.g. absence of, partial or full integration) is absent from the literature, and this exploratory study presents a framework proposal to address that gap.

Huntley (1998) also pointed out that the lack of integrated science and mathematics activities could be a consequence of barriers to teaching using an integrated approach, and PBL could play a major role in this respect. Some researchers have highlighted the process of asking questions and conducting investigation, such as PBL, as one of the best approaches to teaching science in interdisciplinary ways (Bryan et al., 2016; Hmelo-Silver, 2004; Zemelman et al., 2005).

### **Problem-Based Learning in Science and Mathematics Initial Teacher Education**

One goal of science and mathematics integration is to foster students' ability to use skills from both areas in problem-solving activities (Furner & Kumar, 2007; McHugh et al., 2018; Mason, 1996). PBL could be a successful instructional approach to promote integration (Borhan, 2014; Furner & Kumar, 2007). Brears et al. (2011) contended that PBL is situated within the domain of inquiry-based learning. However, according to Navy et al. (2021), what differentiates an inquiry lesson from a PBL lesson is that "a PBL lesson must include, by definition both process skills and inquiry" (p. 1491). These approaches are similar to those reported by Borhan (2014), who mentioned that PBL provides opportunities for PSTs to develop skills and acquire knowledge simultaneously. Furthermore, one strong characteristic of the PBL approach is that the activity is organized around a specific authentic problem that must be solved (Navy et al., 2021). Since the task is founded on inquiry, the answer to the problem must be unknown to the students in advance of the investigation (Navy et al., 2021).

Problem-solving in real-life science situations, or situations related to the students' context, could be addressed applying mathematics (Davidson et al., 1995). However, some studies identify difficulties in this process. For example, Navy and Kaya (2020), in a study with PSTs, found that mathematics was the content area least integrated with science in the integrated units they designed in PBL contexts.

Although several studies have showed the effectiveness of PBL as an instructional approach for K-12 learning (Navy & Kaya, 2020), few studies have focused on PBL in initial teacher education (Borhan, 2014; Navy & Kaya, 2020; Navy et al., 2021). It is important to understand how PSTs address problem-solving and to evaluate PSTs' performance in problem-solving (Son & Lee, 2020). In their practices, PSTs could be particularly anxious about implementing open-ended teaching strategies (Bencze, 2010), such as interdisciplinary PBL. However, research results show that the educational approach of PBL can increase PSTs' content knowledge (Borhan, 2014), pedagogical content knowledge, critical thinking skills (Borhan, 2014; Senocak et al., 2007), problem-solving skills (Borhan, 2014; De Simone, 2008), self-direct learning (Borhan, 2014; Senocak et al., 2007), cooperative learning (Borhan, 2014; Brears et al., 2011; Senocak et al., 2007) and reflective practice (Brears et al., 2011). In fact, the study conducted by Brears et al. (2011) suggests PBL is an effective strategy to support inquiry of an interdisciplinary nature. Moreover, it is vital to understand which are the most effective practices to foster PSTs' confidence about PBL. Wang (2020) has concluded that inquiry-based pedagogical instruction can be a promising alternative to direct instruction in science teacher education. Along this line of thought, some studies focused on teachers' or PSTs' development of PBL units as learners (e.g. Navy & Kaya, 2020; Pepper, 2013). Pepper (2013) noted an increase in PSTs' perceptions about their confidence in teaching science research skills as a result of their engagement with the development of PBL units. Navy and Kaya (2020) also concluded that the development of PBL units can be used as an educational strategy to integrate STEM education in initial teacher education contexts, despite some barriers perceived by PSTs. The role of PBL in teacher education is also highlighted by An (2017) study, concluding that inquiry-based knowledge construction processes could help PSTs improve their science and mathematics interdisciplinary instructional designs. All these studies stress the importance of PSTs experiencing the PBL approach in their teacher education programmes. This idea is reinforced in Navy et al. (2021) study, in which it is suggested that teacher educators should focus on helping new teachers enact PBL in their classroom.

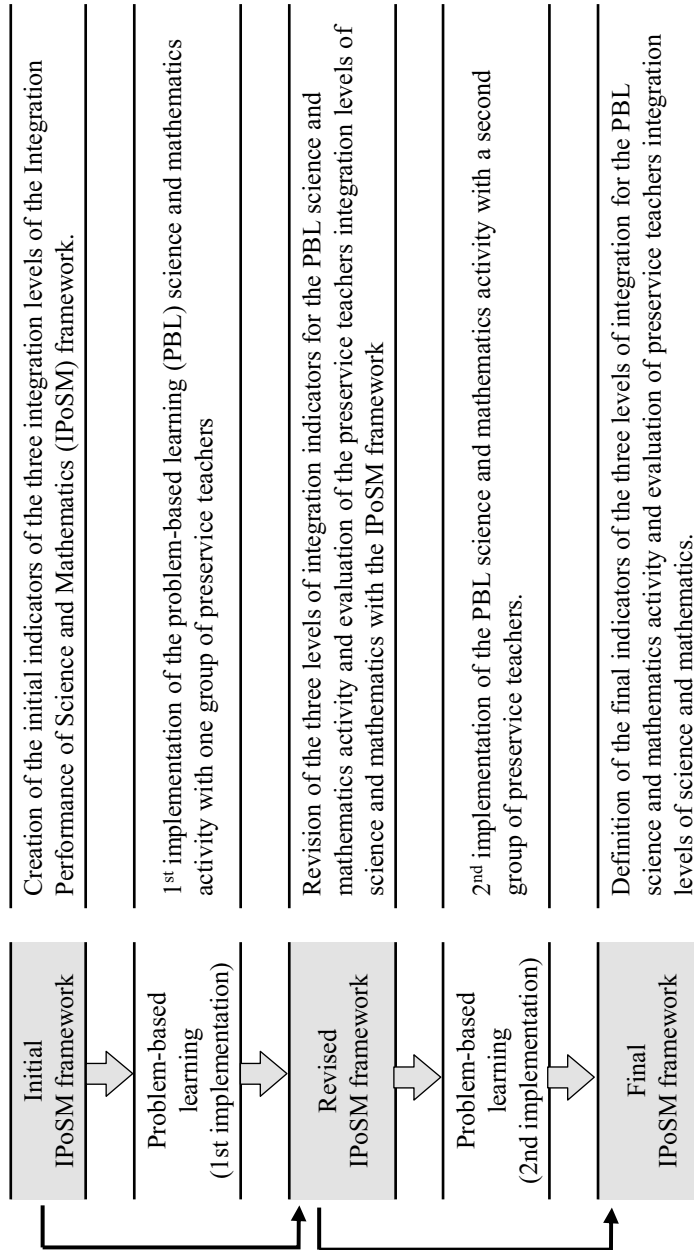
Focusing on PSTs' science and mathematics integration development, in this study an interdisciplinary PBL activity was proposed to PSTs, which is described in the "5" section. Integration in this study is in the context of STEM but focuses particularly on the integration of science and mathematics.

## Research Methodology

### Research Design

This study uses qualitative methods of data collection and data analysis to identify and describe the level of integration of science and mathematics that PSTs achieved in their work on a PBL activity. The research design (Fig. 1) focuses on the PSTs' ideas on the different phases of the PBL activity, titled *STEM Bees*, to identify the PSTs' science and mathematics integration level in each phase.

The research design is structured on the implementation of the integrated PBL activity in two moments corresponding to two academic years. The PSTs worked with the same activity in the first and second implementation.



**Fig. 1** Research design proposal

## Participants and Context

Participants were senior PSTs of a 3-year initial teacher education programme from a Portuguese higher education institution. In this program, PSTs had to attend general education subjects as well as specific content subjects, such as Portuguese, arts, mathematics, science, history and geography. The programme also includes introduction to the didactics of these subjects and short internships. These particular PSTs were being prepared to work in kindergarten or elementary schools (6-12-year-old students) and already had field experience within those contexts in four 2-week internships. To achieve teaching professionalization, they needed to obtain a master's degree. In the first moment of implementation, 41 PSTs participated and, in the second one, 13 PSTs participated, organized in small groups. All the participants were female, except one. Their age ranged mainly from 19 to 21 years and the oldest participant was 49 years old.

The article focuses on the second implementation, in which each group was identified with a number between 1 and 4 (PSTs1 to PSTs4). A letter was added to identify a specific PST within the group (e.g. PST1a, PST1b). The participants had already coursed five mathematics subjects (Introduction to Theory of Numbers; Numbers and Operations; Statistic and Probabilities; Algebra and Functions; Geometry, Quantities and Measures) and two science subjects (Chemistry and Physics; Human Biology and Health) during their program. They had previous experiences of integration in science and mathematics subjects in which teachers usually collaborate, such as Chemistry and Physics as well as Introduction to the Theory of Numbers, Human Biology, and Health and Statistic and Probabilities. In the last semester of their programme, they experienced strong integration between the subjects of Earth and Life Sciences, and also Mathematics Modelling. Some activities implemented within the collaboration context of those subjects followed a PBL approach.

The research team was constituted by two science and two math teacher educators. The *STEM Bees* activity was created collaboratively by one science and one math teacher educator from the research team and revised by the other ones. It was implemented in a science course related to evolution and the origin of the Earth and life, taught by one science teacher with the collaboration of one math teacher in the team.

## Design of the PBL *STEM Bees* Activity

The *STEM Bees* activity was designed with an integrated focus using bees and their products as study object, articulating with geometry and measurement, which provided the mathematical dimension. According to Navy et al. (2021), recent uses of PBL involve it serving as an approach to integrate both content and methods. Therefore, the *STEM Bees* activity enabled content integration (Davidson et al., 1995), mobilizing existing curriculum goals from science and mathematics of the PSTs' initial teacher education programme, and process integration (Davidson et al., 1995; Koirala & Bowman, 2003), combining the problem-solving processes of science and mathematics to solve the PBL research problem.

PBL is situated within the domain of inquiry-based-learning (Brears et al., 2011) and includes both process skills and inquiry (Navy et al., 2021). Therefore, the *STEM Bees* activity was organized in an inquiry cycle within an Inquiry Learning Space (ILS) on the GO-Lab sharing and authoring platform (Cavadas & Branco, 2020). The ILS followed an inquiry cycle structured under the five phases proposed by Pedaste et al. (2015): Orientation, Conceptualization, Investigation, Conclusion and Discussion. It was designed according to research suggestions for effective integrated pedagogy, by allowing PSTs to solve problems using a variety of problem-solving strategies that lead to legitimate answers (An, 2017). One advantage of ILS is that it allows students to input written, audio and visual productions that can be used for subsequent teacher analyses and feedback.

Table 1 presents the organization of the PBL activity's phases and tasks. The proposal has a global research problem divided into two problems focused on the hexagonal shapes of the alveoli produced by bees (problem 1) and on structure of the honeycomb with a 3-dimensional model of the alveolus (problem 2). The activity was created by the research team based on the one presented by Teles et al. (1997). Nazzi's (2016) study was used as a theoretical reference to support the observation that "hexagons possess the highest surface/perimeter ratio, compared to other polygons that can be used for tiling the plane, suggested that honeybees build their hexagonal cells in order to achieve the best economy of material" (p. 1).

Concerning the research problem, efficiency is considered a parameter that gives bees an evolutive advantage, such as less consumption of energy and material to construct alveoli, and therefore a better survival and reproductive performance. PSTs were not informed a priori of the meaning of efficiency because it was intended that they think about it that during the PBL activity.

### Implementation of the PBL STEM Bees Activity

Each *STEM Bees* implementation moment with different PST groups involved 2 weeks of co-teaching by one science and one mathematics teacher educator in online environments. The online teaching context is justified by the confinement caused by the COVID-19 pandemic. Each implementation moment included four online live lessons with PSTs and tasks done within the *STEM Bees* ILS environment, as follows:

**Session 1** The teachers explained what an ILS is, detailed its features and organized the PSTs into small groups (three to four students). It was explained that all productions should be written or dropped in the group's specific inquiry learning space. After that, PSTs were enrolled by their teachers in the orientation phase, in which they explored a text about evolution, solved exercises about bees' anatomy, behaviour and an inquiry-based learning activity about pollination and bees' role in biodiversity sustainability (Orientation). In the sequence, the science and mathematics integrated context of the *STEM Bees* activity and the global research problem were presented by teachers. Teachers introduced problem 1 to PSTs and globally presented the tasks that each group of PSTs should accomplish.

**Autonomous Work 1** Each group had 1 week to solve problem 1. PSTs had to identify initial data, create hypothesis (Conceptualization 1), define and implement a plan, collect and interpret data (Investigation 1) and present their conclusions (Conclusion 1).

**Session 2** The discussion 1 phase was organized in online sessions with two groups each. The PSTs' work on problem 1 was presented by them and discussed with their peers and teachers. More than one discussion session occurred, depending on the number of groups. At the end of this session, problem 2 was introduced.

**Autonomous Work 2** PSTs had to follow a process similar to the previous one to solve problem 2 (Conceptualization 2, Investigation 2 and Conclusion 2).

**Session 3** PSTs' work on problem 2 was presented and discussed (Discussion 2) and, finally, PSTs were prompted to find relationships and draw a global conclusion for the initial research problem.

### Data Collection

The data collection was organized in two moments corresponding to each implementation of the *STEM Bees* activity. In each moment, data for this study included the work of the PSTs' groups recorded on

the ILS in each task of the PBL activity. The data collected on the ILS included written statements and visual productions. The interactions between the PSTs groups and the teachers during the discussion phase were video-recorded and then transcribed.

## Data Analysis

Data analysis was based on the framework proposed by Huntley (1998) about connections between science and mathematics in classroom practice. The authors drew upon Huntley (1998) matrix of intra-disciplinary, interdisciplinary and integrated education to propose a new framework, intended as a basic support structure for identifying PSTs' integration performance of science and mathematics (IPoSM) with three levels, using the same symbols proposed by the author (Table 2).

Level 1 represents absence of integration. PSTs do not present connections between science and mathematics; their products or ideas only express ideas of science or ideas of mathematics.

Level 2 represents partial integration. At this level, connections between science and mathematics are present; however, there is an uneven connection expressed by using more ideas from one area than the other. At level 2, we can also find infusion and transfer opportunities for connecting mathematics and science (Baxter et al., 2014). Regarding infusion connection, there is mathematics problem-solving related to science concepts, and in transfer connection, science inquiry is supported by mathematical processes and concepts (Baxter et al., 2014).

Level 3 represents full integration of science and mathematics, making it difficult to distinguish a dominant area because both play a similar role.

The research team created the initial indicators in each level for the *STEM Bees* activity based on the literature (AMSE et al., 2016; Nazzi, 2016; Teles et al, 1997), anticipating the absence of relationships or the existing relationships among the science and mathematics contents. After the first implementation of the *STEM Bees* activity with the initial group of PSTs, the original indicators were revised by the research team to better express the three levels of integration, due to further reflection on the teacher educators' part and new insights resulting from the PSTs' work. Some new weak and meaningful connections between ideas of science and mathematics, as well as within one subject, emerged from the analysis of the PSTs' work and were included in the framework. The indicators were as detailed as possible to ensure the validity of the three integration levels and coding reliability.

In a second implementation moment with another group of PSTs, their integration performance was analysed. To ensure coding reliability of the PSTs' performance, the authors were divided into two research teams constituted by a science teacher and a mathematics teacher.

The PSTs' productions in the different phases (conceptualization 1 and 2, investigation 1 and 2, conclusion 1 and 2, and global conclusion) were recovered on the ILS that each PSTs' group filled with their answers and reasoning. Each research team analysed the PSTs' productions for each phase separately, classified them according to one of the three levels of the IPoSM framework, using the indicators to include those productions in each level. Afterwards, both research teams met to discuss the results of the PSTs' performance and achieve a consensus of the integration levels obtained, ensuring reliability. This was an inductive and recursive process because the classification of the PSTs' productions according to one of the three levels of the IPoSM framework helped refine the indicators, which led to the creation of the final indicators of the three levels of integration (see 23).

## Findings

PSTs' level of integration of science and mathematics regarding the second implementation of PBL activity *STEM Bees* is presented in this section. Figure 5 summarizes the presentation of the findings. In a first moment, the level of integration achieved by the PSTs groups in each PBL phase is presented,

according to the indicators of the IPoSM framework. In a second moment, specific examples of PSTs' productions are used to clarify the level of integration achieved and the specific indicators associated.

Table 3 summarizes the level of integration of each PBL phase analysed, as achieved by each PST group.

The PSTs' performance on PBL phases of problem 2 showed higher integration levels than with problem 1. In addition, level 3 integration was commonly achieved in the global conclusion. Only one group (PSTs1) did not reach level 3 of integration in problem 1 and problem 2 phases, although they achieved it in the global conclusion. All the other groups reached level 3 at some point of the PBL phases. PSTs2 and PSTs3 achieved level 3 of integration in investigation 2 but not in investigation 1.

In the next sections, the PSTs' work and the ideas they shared in the discussion phases are presented to better clarify their integration performance in the PBL phases related with problems 1 and 2 and global conclusion about the initial research problem.

### Problem 1

In the conceptualization phase, PSTs identified the most relevant data and drew up hypothesis about problem 1. Table 4 presents examples of PSTs' work related with the two levels of integration identified.

Although PSTs1 mentioned the volume of the alveolus in the identification of initial data about problem 1 in the conceptualization phase, this is not a requirement to solve the two-dimensional problem 1.

PSTs only presented level 2 of integration at the investigation phase of problem 1, in their written work on the ILS. However, when presenting their investigation in the discussion phase, some PST groups improved their level of integration to level 3 (Table 5).

Levels 2 and 3 of PSTs' integration performance were identified in the conclusion phase of problem 1 (Table 6). In particular, PSTs3 showed a level 3 of integration in that phase, but in the discussion session they mobilized knowledge from previous phases to better express and develop their conclusions.

### Problem 2

PSTs achieved level 2 and level 3 of integration in their answers in problem 2 conceptualization phase (Table 7):

The three levels of integration were identified in PSTs' work about the investigation phase of problem 2, considering the planning, implementation and data interpretation (Table 8).

The PSTs' work achieved level 1 and level 3 of integration in the conclusion phase of problem 2 (Table 9).

### Global Conclusion

In the global conclusion, the PSTs achieved level 2 and 3 integration (Table 10).

### Discussion

The *STEM Bees* activity is in line with Frykholm and Glasson (2005) suggestion that effective and integrated science and mathematics education should contemplate a less fragmented, more relevant and stimulating experience for learners. The work done by the PSTs on the *STEM Bees* activity evidenced that mathematics benefited from the context of science, giving meaning to ideas such as polygon area and perimeter, tessellation using regular polygons, volume and area of geometric solids. On the other hand, the data produced by mathematics reasoning contributed to foster PSTs' grasp of the reasons why the

hexagonal shape was selected for the alveoli, and which was the best three-dimensional form to construct honeycombs, connecting this with evolution notions. Within their work, the PSTs used representations of science and mathematical concepts in multiple forms, including visually, with symbols, words and numbers, which are problem-solving strategies that could emerge from interdisciplinary approaches, such as mentioned by An (2017).

Some integration indicators which resulted from the first moment of implementation of the *STEM Bees* activity were not expressed by the PSTs in the second moment of implementation. This is quite normal because different PSTs have different approaches to solve the problems proposed. It should be clarified that a team of science and mathematics teacher educators applying the *STEM Bees* activities to a new group of PSTs could encounter different productions which may not exactly correspond to the indicators presented in the 23. Therefore, the framework presented in the 23 should not be perceived as a final product, but a product in development to which new indicators could be added by new applications of the *STEM Bees* activity. This strengthens the need for collaborative work between science and mathematics teachers to further improve the science and mathematics integrated activity and the indicators used to identify PSTs' integration performance.


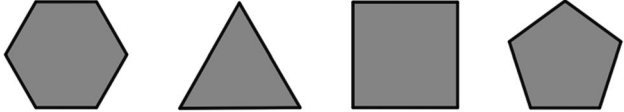
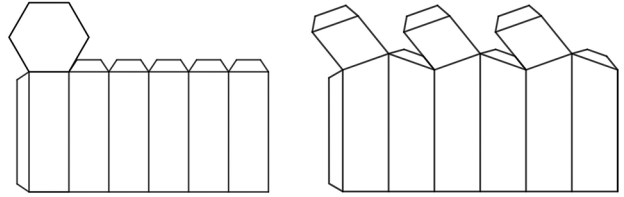
Nevertheless, those productions could be framed in the integration levels presented in Table 2 and added to the indicators of each level of the IPoSM framework.

Although this was not the focus of this study, the PSTs' work on the PBL activity and their performance in the discussion phase suggested they also developed other competencies mentioned in the literature about PBL, such as increased content knowledge (e.g. Borhan, 2014), critical thinking skills (e.g. Borhan, 2014; Senocak et al., 2007), problem-solving skills (e.g. Borhan, 2014; De Simone, 2008), self-direct learning (e.g. Borhan, 2014; Senocak et al., 2007), cooperative learning (e.g. Borhan, 2014; Brears et al., 2011; Senocak et al., 2007) and science inquiry competencies (e.g. Chowdary et al., 2014).

### Level 1 Integration

It was noted that some PST groups experienced some tension with integration because a few topics about the problems seemed difficult for them to integrate, a perception also discussed in other works (e.g. Koirala & Bowman, 2003). In terms of the PSTs' performance on the integration of science and mathematics regarding *STEM Bees*, the findings suggest that some PSTs struggled with the connections to the biological context of the problem and the definition of a strategy to address it. Because of these difficulties, some PSTs only focused on mathematics ideas to tackle the problem and did not transpose these ideas into the bees' work, achieving only level 1 in some PBL phases. This finding confirms previous research (Lederman & Lederman, 2013) that, in contexts of problem solving, when students are expected to arrive at solutions, they tend to prioritize the conventions of either science or mathematics. Moreover, as shown by Son and Lee (2020), PSTs tend to focus on straightforward steps to reach right answers. The outcome of this process is that one of the disciplines is given primacy over the other, a result opposed to the conceptualization of true integration (Lederman & Lederman, 2013). This conventional approach to a problem is evidence of the relevance of more PBL experiences in initial teacher education (Borhan, 2014; Navy & Kaya, 2020; Pepper, 2013) with a focus on integration. Consequently, this work suggests that during integrated PBL approaches, teacher educators should pay more attention to PST groups who reveal a lower integration level since the beginning of their work on the PBL activity, acting as facilitators and helping them achieve higher levels of integration.

**Table 1** Organization of the PBL activity *STEM Bees*

Phases	Task
Orientation	Read an introduction to evolution. Explore bees' anatomy and behaviour. Resolve an inquiry-based learning activity about pollination and bees' role in biodiversity sustainability
Research Problem	Present initial ideas about the global research problem: <p>“The alveoli elaborated with beeswax are geometrically similar. Usually, the alveolus has the shape of a regular polygon, in this case a regular hexagon, as you can see in Fig. 2.”</p>
	
<p><b>Fig. 2</b> Alveoli</p> <p>For a better understanding of bees' behaviour, we suggest that you answer this research problem:  <b>“Why is the shape of the alveoli in beehives efficient?”</b></p>	
Conceptualization 1	Analysis of problem 1: <b>“What reasons explain why the shape of an alveolus is hexagonal and not a triangle, a quadrilateral or a pentagon?”</b> , identification of initial data and hypothesis definition (Fig. 3)
	
<p><b>Fig. 3</b> Polygons</p>	
Investigation 1	Planning and experimentation, data collection, data interpretation
Conclusion 1	Find relationships and draw a conclusion for problem 1
Discussion 1	Share and discuss the explanations with teachers and other PSTs
Conceptualization 2	Analysis of problem 2, identification of initial data and hypothesis definition. <p>“You are going to test two models of solids (Alveolus model 1 and Alveolus model 2) and verify which is more advantageous for the construction of the honeycomb. Begin your work constructing two honeycombs. In one of the honeycombs, you should only use <i>Alveolus Model 1</i> and in the other one use only <i>Alveolus Model 2</i>.”</p>
	
<p><b>Fig. 4</b> Alveolus model 1 and alveolus model 2</p> <p><b>“Which solid is more efficient to construct alveoli and form a honeycomb?”</b></p>	
Investigation 2	Planning and experimentation, data collection, data interpretation
Conclusion 2	Interpret data, find relationships and draw a conclusion for problem 2
Discussion 2	Share and discuss the explanations with teachers and other PSTs.
Global conclusion	Find relationships and draw a global conclusion for the initial research problem

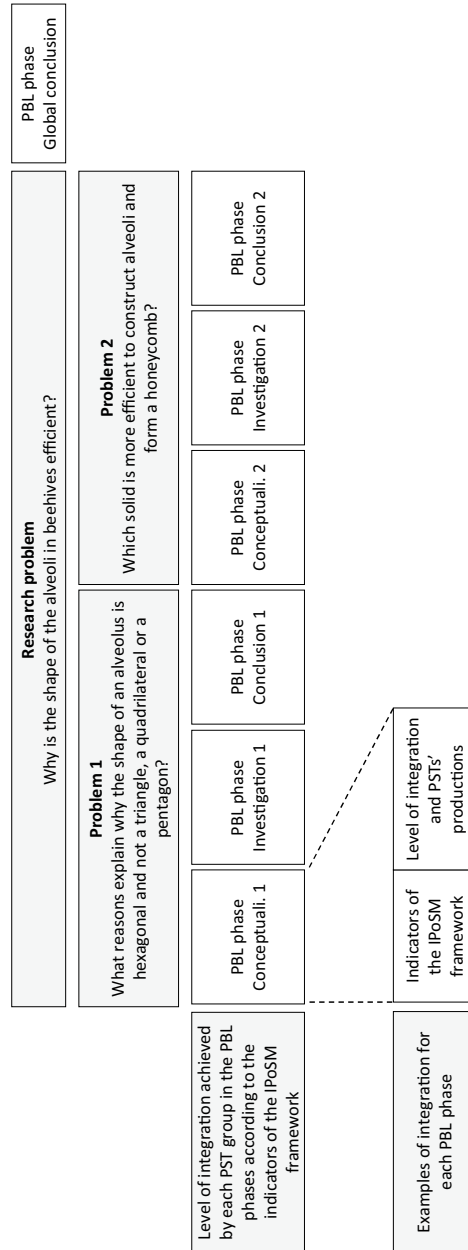

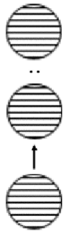


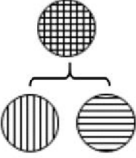


Fig. 5 Organization of the findings

**Table 2** Framework used for identifying PSTs' integration performance of science and mathematics (IPoSM). The symbols and the description synthesize the cognitive process related to each level

Integration levels	Description
Level 1: No integration	  Only ideas from mathematics. Only ideas from science.
Level 2: Partial integration	  Weak connections of mainly mathematics ideas with some ideas of science. Weak connections of mainly science ideas with some ideas of mathematics.
Level 3: Full integration	 Meaningful connections of ideas from mathematics and science.

Adapted from Huntley (1998)

**Table 3** Levels of PSTs' integration of science and mathematics in the conceptualization, investigation and conclusion phases of the PBL activity *STEM Bees*

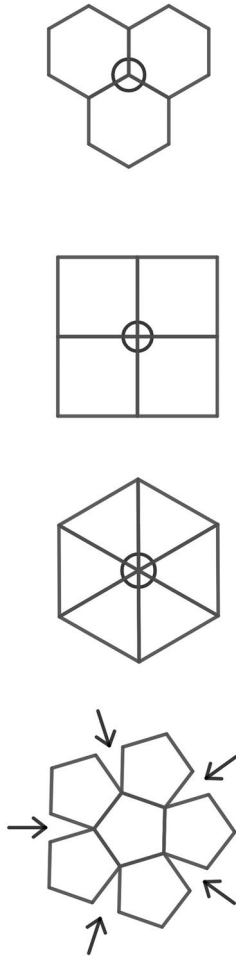
PSTs group	Problem 1			Problem 2			Global Conclusion
	Concept. 1	Inv. 1	Concl. 1	Concept. 2	Inv. 2	Concl. 2	
PST1							
PST2							
PST3							
PST4							

**Table 4** Integration levels of PSTs' work on the conceptualization phase of *STEM Bees* problem 1

<b>Levels of integration and PSTs' productions</b>	
<b>Level 1</b>   Only ideas from mathematics:	
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">perimeter-to-area ratio</div>	<p>“The data that we consider relevant for the resolution of the problem are the perimeter, the area and the volume. Indeed, the shape of the base of the alveoli could be a triangle, a quadrilateral or a pentagon. However, the hexagonal shape should be the best one because its area is the largest.” (PSTs1; Conceptualization 1)</p>
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">Polygon with the biggest area for the same perimeter</div>	
<b>Level 3</b>   Meaningful connections of ideas from mathematics and science:	
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">The alveoli must fit together without gaps between them</div>	<p>“If bees need to store as much honey as possible with a minimal expenditure of wax, then the pattern should not allow for empty spaces or overlaps, in order to optimize the process . . . If the wax expenditure is the same and the storage capacity is different, then the shape of the alveoli that corresponds to the polygonal base with the largest area will be more efficient.” (PSTs2; Conceptualization 1)</p>
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">Regular tessellation of a plane surface with regular polygons</div>	
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">Each alveolus in the honeycomb share their walls with others alveolus</div>	
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">Polygon with the biggest area for the same perimeter</div>	

**Table 5** Integration levels of PSTs’ work in the investigation phase of *STEM Bees* problem 1

Indicators	
<p>Regular polygons</p> <p>Regular tessellation of a plane surface with regular polygons</p> <p>The alveoli must fit together without gaps between them</p> <p>Polygon with the biggest area for the same perimeter</p> <p>Polygon with the smaller perimeter for the same area</p> <p>Using minimum material (beeswax) and energy consumption</p>	<p>Level 2   Weak connections of mainly mathematics ideas with some ideas of science:</p> <p>“To solve this problem, we will start by drawing patterns with the different geometric shapes (triangle, quadrilateral, pentagon, and hexagon), to determine which allow to form a pattern without white empty spaces or overlaps. Then, starting from a base figure (to illustrate that the wax expenditure is the same), we will “build” the geometric shapes that form a regular fit pattern. As the volume depends on the area of the base polygon, the next step comprises the calculation of that area for each of the figures. Finally, we will draw up our conclusions.” (PSTs2; Investigation 1)</p>
<p>Using minimum material (beeswax) and energy</p> <p>Perimeter-to-area ratio</p> <p>Each alveolus in the honeycomb share their walls with others alveolus</p> <p>Polygon with the biggest area for the smaller perimeter, for the same area</p> <p>Store more honey with the same expenditure of material</p>	<p>Level 3   Meaningful connections of ideas from mathematics and science:</p> <p>PST2a: Our idea is... If bees need a prism that allows them to store honey with the least amount of wax possible, as we have already seen that they use the hexagon, so we ask why the hexagon, and not the other two [triangle or square]? What distinguishes it? ... As volume of polygon depends on base area measure, we must calculate the area of each polygon to understand why the hexagon... [after presenting the calculations] The hexagon’s area is 23.4 cm<sup>2</sup>, that is, the largest area among the three polygons. With the assumption that if all shapes fit without gaps or overlaps, we assume that the wax expenditure would be the same for the three... Science TE: Did they have as control variable the same perimeter in all figures, which is 18? PST2a: Yes, maintaining this perimeter was our way of expressing that there would be an equal wax consumption for every shapes Math TE: State clearly why hexagon is the most efficient shape PST2c: It has the shape that has a greater volume so it will be able to store a greater amount of honey with a minimum amount of wax Math TE: With the same, in this case with the same. (PSTs3; Discussion 1)</p>



**Fig. 6.** Tessellation of a plane surface with regular polygons done by PSTs4

**Level 3 |** Meaningful connections of ideas from mathematics and science:

PST2a: Our idea is... If bees need a prism that allows them to store honey with the least amount of wax possible, as we have already seen that they use the hexagon, so we ask why the hexagon, and not the other two [triangle or square]? What distinguishes it? ... As volume of polygon depends on base area measure, we must calculate the area of each polygon to understand why the hexagon... [after presenting the calculations] The hexagon’s area is 23.4 cm<sup>2</sup>, that is, the largest area among the three polygons. With the assumption that if all shapes fit without gaps or overlaps, we assume that the wax expenditure would be the same for the three...

Science TE: Did they have as control variable the same perimeter in all figures, which is 18?

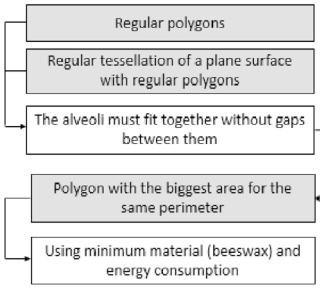
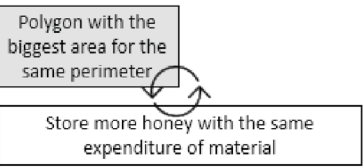
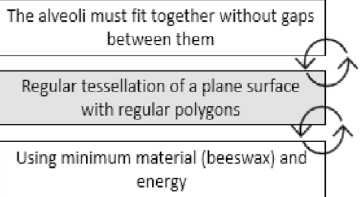
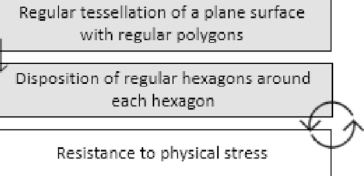
PST2a: Yes, maintaining this perimeter was our way of expressing that there would be an equal wax consumption for every shapes

Math TE: State clearly why hexagon is the most efficient shape

PST2c: It has the shape that has a greater volume so it will be able to store a greater amount of honey with a minimum amount of wax

Math TE: With the same, in this case with the same. (PSTs3; Discussion 1)

**Table 6** Integration levels of PSTs' work in the conclusion phase of *STEM Bees* problem 1

Indicators	Levels of integration and PSTs' productions
	<p><b>Level 2</b>   Weak connections of mainly mathematics ideas with some ideas of science:</p> <p>“Three of the geometric shapes could be used and all of them would unify the alveoli. However, the square would occupy a smaller area compared to the hexagon. In the triangle, bees would have to build more cells than in a hexagon. The hexagonal shape is chosen because, for the same material used (wax), it is possible to obtain a larger available space, being that triangles, quadrilaterals and hexagons are the only regular polygons that promote a mosaic style pattern without gaps, through its grouping.” (PSTs1; Conclusion 1).</p>
	<p><b>Level 3</b>   Meaningful connections of ideas from mathematics and science:</p> <p>“The shape of the base is hexagonal because it presents a larger area in relation to the other geometric shapes. Using the same amount of wax, that is, starting from the same base figure with the same perimeter, the shape of the most efficient honeycomb for storing the largest amount of honey would be the one with the largest area of the base polygon, that is, the hexagonal. (PSTs2; Conclusion 1)</p>
	<p>“To make [alveoli with] triangles, bees would have to work harder and also use more wax to build them. . . Thus, we conclude that the regular hexagon is the best geometric figure to occupy the total area of the hive, with less waste of space and wax, since they will have to make fewer alveoli. (PSTs4; Conclusion 1)</p>
	<p>PST2a: When we join hexagons, they form a 360° angle (120° each). With the least expenditure of material, the hexagon is the most mechanically stable structure. If we apply force to one side, the other two respond with identical but opposite forces . . . In a honeycomb, by combining several hexagons, the structure is strengthened. (PSTs2; Discussion 1)</p>

## Level 2 Integration

The tension in integration mentioned by Koirala and Bowman (2003) also resulted in productions classified on level 2. PSTs' productions showed a predominance of mathematics ideas, mainly in problem 14. This could be related to the specific context of each problem and some difficulties in understanding it, a situation also reported by Frykholm and Glasson (2005) that may compromise PSTs' problem-solving ability. Moreover, problem 1 has more proximity to an infusion connection type (Baxter et al., 2014), focused on math problem solving related to science concepts, and this could be one reason for the poor expression of science ideas in PSTs' work addressing this problem. A mathematical proof can lead to a correct mathematic answer (Baxter et al., 2014) but that may not be enough if the scientific context where it emerged is forgotten. In fact, on problem 1, one group whose focus was mainly on mathematics

**Table 7** Integration levels of PSTs' work in the conceptualization phase of *STEM Bees* problem 2

Indicators	Levels of integration and PSTs' productions
	<p><b>Level 2</b>   Weak connections of mainly mathematics ideas with some ideas of science:</p> <p>“The relevant data for solving the problem is the volume of the alveolus (amount of honey that can be inserted in it). In the model of alveolus 1, the volume of honey stored is smaller, while in the model of alveolus 2, the volume will be greater because the base is formed by 3 lozenges, making the alveolus a little more "spacious" because its base is not flat.” (PSTs1; Conceptualization 2)</p>
	<p><b>Level 3</b>   Meaningful connections of ideas from mathematics and science:</p> <p>“If bees need to store the largest amount of honey with a minimum expenditure of wax, then the most efficient honeycomb will be the one that, for a given volume, has the smallest area. If bees want the well-being and durability of the hive, then they should opt for a structure that guarantees its resistance and flexibility.” (PSTs2; Conceptualization 2)</p>

**Table 8** Integration levels of PSTs' work in the investigation phase of *STEM Bees* problem 2

Indicators	Levels of integration and PSTs' productions
3D elements characteristic of each alveolus model	<p><b>Level 1</b>   Only ideas from mathematics:                      “An adequate plan for solving the problem would be to determine the volume of the two models as well as their construction, thus facilitating understanding... to fill the two models with the same material (sand) and determine the weight placed on the two available models.” (PSTs1; Investigation 2)</p>
Volume of each alveolus model	<p><b>Level 2</b>   Weak connections of mainly mathematics ideas with some ideas of science:                      “As can be seen [Fig. 7], the area of model 2 is smaller compared to the area of model 1... consequently, they take less wax to occupy the same space.” (PSTs4; Investigation 2)</p>
Surface area of each alveolus model	<p><b>Modelo 1</b></p>
Using minimum material (beeswax) and energy	<p>Área total = Área dos retângulos + Área do hexágono</p> <p>Área de cada retângulo:</p> $A = c \times l$ $A = 9 \times 3$ $A = 27 \text{ cm}^2$ <p>Como no modelo 1 há 6 retângulos então: <math>27 \times 6 = 162 \text{ cm}^2</math></p> <p>Área do hexágono regular:</p> $\frac{b \cdot a}{2} \times 6 = \frac{3 \times 2,6}{2} \times 6 = 23,4 \text{ cm}^2$ <p>Área total = <math>23,4 + 162</math></p> <p>Área total = <b>185,4 cm<sup>2</sup></b></p>
Using minimum material (beeswax) and energy	<p><b>Modelo 2</b></p>
Surface area of each alveolus model	<p>Área total = Área dos trapézios + Área dos losangos</p> <p>Área de cada Losango:</p> $\frac{D \times d}{2} = \frac{5 \times 3,5}{2} = 8,75 \text{ cm}^2$ <p>Como no modelo 2 há 3 losangos então: <math>8,75 \times 3 = 26,25 \text{ cm}^2</math></p> <p>Área de cada Trapézio:</p> $\frac{B + b}{2} \times a = \frac{9 + 8}{2} \times 3 = 25,5 \text{ cm}^2$ <p>Como no modelo 2 há 6 trapézios então: <math>25,5 \times 6 = 153 \text{ cm}^2</math></p> <p>Área total = <math>26,25 + 153</math></p> <p>Área total = <b>179,25 cm<sup>2</sup></b></p>

**Fig. 7** Calculation of the total surface area of alveolus model 1 and model 2, done by PSTs4

**Level 3** | Meaningful connections of ideas from mathematics and science:

“To solve the problem, we will start by comparing the volume of both models, using rice berries (empirical approach), to understand if there are differences in the amount of honey that each one would be able to store. Then, the area of each model will be calculated, in order to determine which needs the least amount of wax.” (PSTs2; Investigation 2)

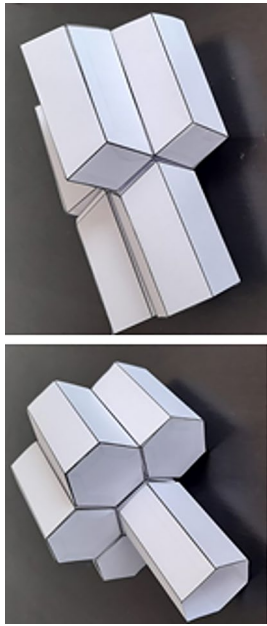
**Indicators**

3D elements characteristic of each alveolus model

Structures physically strong, leak-proof, resistant to predators' attack and well-suited to the various purposes for which it were built

**Levels of integration and PSTs' productions**

“The last step comprises the complete construction of the combs in order to investigate, through the observation of the structure, the one that will provide greater resistance and solidity to the hive, as well as the reasons for this. [Fig. 8]” (PSTs2; Investigation 2)



**Fig. 8** Examples of PSTs2's productions about model 1 (left image) and model 2 (right image) of alveoli PST3d: “On the question of stability, we reflected about which of the two models is the more stable, the harder, the more resistant. In model 1 you can imagine a straight line dividing the two layers [at the junction of the closed bases of each of the layers]. In the second model the line is already a “zig-zag” that strengthens this structure. In other words, it is easier to break the structure of model 1 than the structure of model 2 due to an impact.” (PSTs3; Discussion 2)

**Table 9** Integration levels of PSTs’ work in the conclusion phase of *STEM Bees* problem 2

<b>Levels of integration and PSTs’ productions</b>	
<b>Level 1</b>   Only ideas from mathematics:	
3D elements characteristic of each alveolus model	<p>“With this reflection it is possible to see that, despite the optical illusion created by the lozenges, this model is only truncated, causing the “open” spaces to be compensated by the three lozenges. Thus, obtaining the same volume for both models . . . Even so, the fact that the alveolus model 2 is truncated and is shorter if we place it side by side and draw a straight line from model 1, does not prevent it from having the same volume. This is because the lozenges end up compensating for the “missing height”. All of this was proven by placing the same load capacity on both models and verifying that their weight was the same.” (PSTs4; Conclusion 2)</p>
Volume of each alveolus model	
<b>Level 3</b>   Meaningful connections of ideas from mathematics and science.	
Using minimum material (beeswax) and energy	<p>“Through the empirical approach, we can see that both models have the same volume, and it is possible to fill them with the same amount of rice. Therefore, the most efficient, in this case, will be the one that requires less wax. When we calculate the areas of each model, we realize that it will be model 2. The honeycomb built with model 1 allows you to imagine a straight line that easily separates both layers. When we look at the honeycomb built with model 2, it is not possible to imagine this straight line separating the layers. The format would be in zigzag. So that means that a honeycomb built with model 2 would be more flexible and resistant in the event of an impact, for example.” (PSTs2; Conclusion 2)</p>
Surface area of each alveolus model	
3D elements characteristic of each alveolus model Structures physically strong, leak-proof, resistant to predators’ attack and well-suited to the various purposes for which it were built	

**Table 10** Integration levels of PSTs' work in the global conclusion of *STEM Bees* activity

Indicators	Levels of integration and PSTs' productions
<pre> graph TD     A[Evolution theory] --&gt; B[Using minimum material (beeswax) and energy]     B --&gt; C[Hexagonal structure of each alveolus model]     C --&gt; D[Structures physically strong, leak-proof, resistant to predators' attack and well-suited to the various purposes for which it were built]             </pre>	<p><b>Level 2</b>   Weak connections of mainly science ideas with some ideas of mathematics.</p> <p>“Natural selection is one of the main reasons why bees today resort to the hexagonal shape. This allowed the species of honeybees to optimize their resources to survive. Thus, the main advantages inherent to the hexagonal shape are related to the fact that it provides greater stability and resistance to the combs, as it allows the bees to use the minimum amount of wax to store the maximum amount of honey.” (PSTs3; Global conclusion).</p>
<pre> graph TD     A[Back-to-back extremities of alveoli fit against each other] --&gt; B[Hexagonal structure of each alveolus model]     B --&gt; C[Maximum space inside the alveolus for growing young bees or storing honey]     C --&gt; D[Using minimum material (beeswax) and energy]     D --&gt; E[3D elements characteristic of each alveolus model]     E --&gt; F[Structures physically strong, leak-proof, resistant to predators' attack and well-suited to the various purposes for which it were built]             </pre>	<p><b>Level 3</b>   Meaningful connections of ideas from mathematics and science.</p> <p>“The hexagonal shape allows better use of the space of the alveoli, where the bees keep the honey. The combs are made of wax and, also, the hexagonal shape allows to spend less wax in the construction of its walls, which become common to other alveoli. Thus, the hexagonal shape achieves a greater storage of honey with the least amount of wax possible. We opted for model 2, which consists of lozenges at the base, because we believe it makes the structure more stable, flexible, and resistant in the event of aggression and/or accidents. The hexagonal shape will allow a zigzag fit, that is, the opposite to the fit in a straight line. This cross fit better protects combs from attacks and other problems that may arise, as it makes the entire structure of the hive stronger.” (PSTs; Global conclusion)</p>

did not define one control variable to compare the results for each polygon, such as perimeter or area, considering the constraints of energy and material savings in bees' evolution. Because of that, they presented a conclusion without full integration and an in-depth justification. On problem 2, the PSTs' work on level 2 focused mainly on the calculation of volume and area and less on efficiency and the structure of the honeycomb.

### Level 3 Integration

Level 3 integration was evident in PST2's work. This group initiated with the highest level of integration in the conceptualization phase and maintained it in most of the other PBL phases. However, a PST group starting with a higher level of integration does not necessarily mean it is going to maintain it until the end, such as happened with PST3 in the global conclusion. So, teacher educators should try to identify the conceptual block that is impacting PSTs' integration performance, reflecting on task content and the reasons why a group is showing a lower level of integration in a specific PBL phase. Teacher educators must present the problem clearly as well some insights about the connections between science and mathematics on the problem to improve PSTs' integration performance.

Results also show that PST groups achieved more level 3 integration in problem 2 rather problem 1. We think the main reason for a better PST performance in integration of science and mathematics on problem 2 could be associated to the experience accumulated in integration on the PBL phases of problem 1 and the knowledge gained from the discussion with their teachers and peers. In the discussion phase, PSTs seemed to integrate better the ideas from the previous phases with the purpose of discussing their findings and conclusions about each problem. In fact, this moment allowed clarification of the PSTs' work on each PBL phase. Frequently, PSTs were more skilful at presenting their ideas orally in the discussion phase than in written outputs. Also, the focused questions asked by the teacher educators contributed to assist PSTs in clarifying their reasoning, which proved beneficial for their integration performance and for creating more transparent connections, as suggested by English (2016). This confirms previous research that suggested PST support through feedback is essential for better performance in integration (Koirala & Bowman, 2003) and for sharing solution strategies with an emphasis on justification (Baxter et al., 2014), improving PSTs' content knowledge about the science and mathematics themes and contents addressed with PBL. When discussing findings, it was noted that PSTs' interdisciplinary content knowledge had improved through clarification of their ideas, supported by the teachers' feedback.

Level 3 integration was also commonly reached at the global conclusion phase of the research problem, resulting in interdisciplinary integration as defined by Greef et al. (2017), because PSTs had done the synthesis of their previous knowledge from both areas with new knowledge. It was noticed that at in the global conclusion phase, PST groups mobilized some aspects of the orientation phase that were important for addressing the problem, for example, evolution. This shows the knowledge acquired by the PSTs during a well-structured orientation phase could be mobilized later in other PBL phases. The global conclusion phase was also supported by the previous discussion, so PSTs established a better relation between the problem and the solutions they had found, showing full integration, as exemplified below:

"The current shape of the alveoli and combs is the most favorable one because it has the largest area and, consequently, allows the greatest storage of honey. With the hexagonal shape, the bees use less wax when building the combs and, in this way, they can form a mosaic-style pattern without gaps, thus making the most of the space in the hive." (PSTs4; Global conclusion)

In accordance with other research (e.g. Davidson et al., 1995; English, 2016; Furner & Kumar, 2007), the findings show that a proper integration of mathematics and science can enrich students' learning context through meaningful connections between overlapping concepts and principles from both areas, as the previous excerpts show.

## Limitations and Further Research

One limitation of the design presented in Fig. 2 is that it is very time consuming because it implies at least two implementations of the science and mathematics PBL activity. The first implementation allows detailed indicators of the three levels of integration to emerge, and the second and next ones are meant to apply those criteria to identify PSTs' science and mathematics integration performance.

The creation of the indicators was an inductive process based on the work the PSTs did on the *STEM Bees* activity. The fact that the participating PSTs were being prepared to act as teachers in the first levels of education, and their prior knowledge of science and mathematics content, could decisively influence their efforts to solve the problems of the *STEM Bees* activity, and therefore the creation of the specific indicators. Therefore, it can be argued that some other indicators could have been defined if different teacher educators had had to create them, influenced by different curriculum experiences and their PSTs' prior knowledge. The implication of this situation is that the indicators necessarily need to take into consideration the specific curriculum attended by the PSTs who are experiencing integration. PSTs can only integrate the science they know and the mathematics they understand.

Another limitation of this study is that it focused on the PSTs' performance in one specific activity—*STEM Bees*. However, one advantage of this design is that it allows continuous improvement of the detailed indicators of integration and enables a more in-depth analysis of an integrated activity. These detailed indicators and the IPoSM framework could also be a fertile ground to create other integrated activities.

The design of the study only allowed the identification of group performance, not individual performance. Further research should consider if continuous experience addressing integrated PBL can help PSTs achieve better integration performance in group and individual work.

We also expect, as mentioned in previous research (e.g. Brown & Bogiages, 2019; Koirala & Bowman, 2003), that experiencing integrated tasks can improve teachers' confidence when designing integrated science and mathematics problems for their students. Even though this study advocates that the proposed framework can be used for identifying PSTs' integration of science and mathematics, further work is needed to evaluate if students can develop a rich understanding of these areas through integration, as stated by Huntley (1998). Furthermore, future work could also aim to identify to what extent the PSTs' prior content knowledge on mathematics and science impacted their results.

Future studies can also research if the IPoSM framework can be used for analysing integration of science and mathematics through PBL in other contexts, such as in-service teacher education, or with other students dealing with science and mathematics contexts.

## Conclusions

In this work, PSTs' integration performance of science and mathematics in the different phases of a PBL activity was examined. PSTs were encouraged to research the specific mathematical properties and relationships addressed, as well as their understanding of the science context of this PBL, a development recognised by Huntley (1998) as being able to contribute to integration.

Focusing on the research question of this study, we have identified that PSTs' integration of science and mathematics is more evident in some phases of PBL than in others, and that the conclusion and discussion phases are key to this integration. Although some PST groups struggled with integration at a specific moment, we noticed that the content knowledge that they acquired in all PBL phases was aggregated for the global conclusion, since almost all of them demonstrated higher integration levels of

science and mathematics. Therefore, the present work suggests that a PBL approach can be useful for PSTs to experience integrated situations in science and mathematics.

This study also presents a proposal of a replicable research design (see Fig. 2) that can be used in PST education to address the integration between science and mathematics, helping to fill the gap mentioned by An (2017) that there are no effective and replicable methods in PST education to develop instruction focused on the connections between science and mathematics. It is suggested that this research design could also be used to address integration in other subjects.

Finally, it was concluded that the IPoSM framework used proved to be useful for identifying the level of integration of science and mathematics in PSTs' work in each phase of the PBL activity. Moreover, keeping the IPoSM framework in mind, teachers may be able to elaborate more effective integrated problem-based learning approaches in science and mathematics.

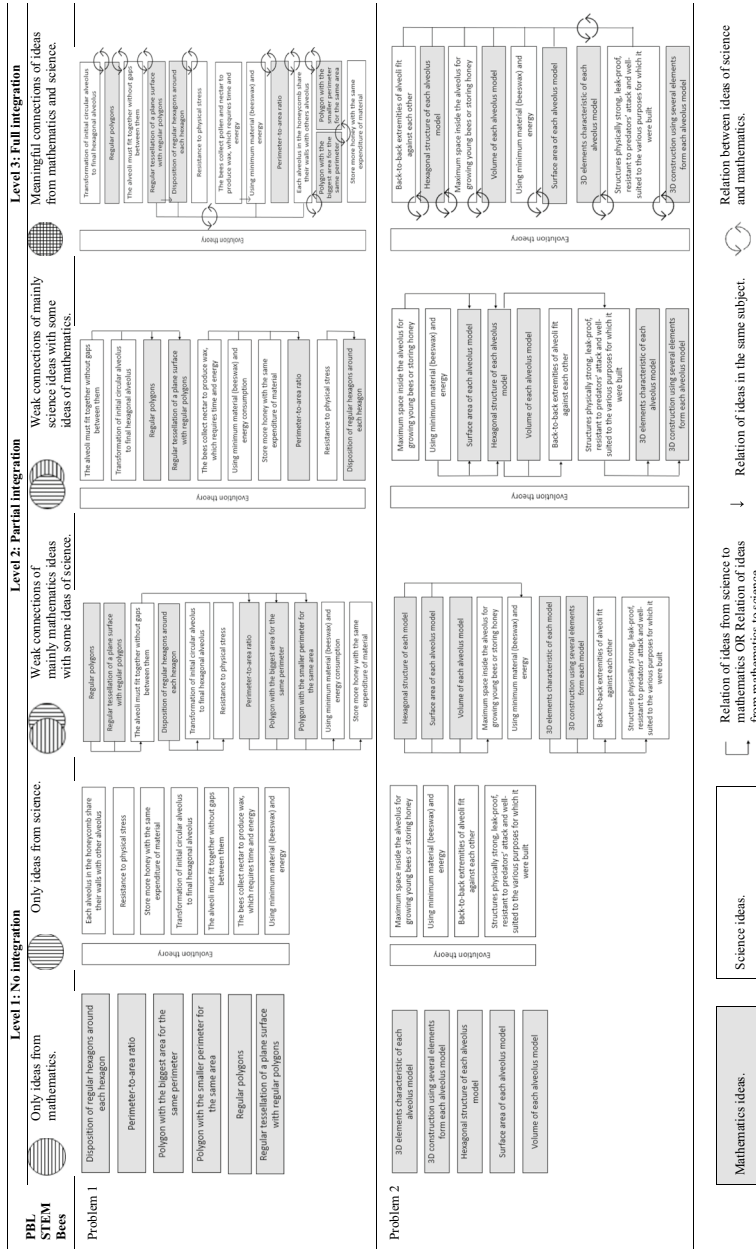
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#### **Declarations**

**Conflict of Interest** The authors declare no competing interests.

## **Appendix**

IPoSM framework with description of the final indicators used to identify PSTs' integration performance of science and mathematics in problem 1 and problem 2 of the PBL *STEM Bees* activity.



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