

Value-at-Risk model based on Extreme Value Theory: Comparison with other models under the Basel Accord

31th International Symposium on Forecasting (ISF2011)

University of Economics - Prague

June 26-29, 2011

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I - Introduction

II - GARCH type models

III - Extreme Value Theory (EVT) models

IV - Comparative study - Before, During and After the GFC

V - Conclusions

I - Introduction

The Basel II Accord requires that banks and other Authorized Deposit-taking Institutions (ADIs) communicate their daily risk forecasts to the appropriate monetary authorities at the beginning of each trading day.

Banks are required to report Value at Risk estimated with a 99% level of confidence to determine regulatory capital requirements.

Basel II accord was designed to reward institutions with superior risk management systems.

Financial Institutions are permitted to use Internal Models to calculate VaR.

Capital requirement are based on VaR and the number of violations:

$$CR_{t+1} = \sup \left\{ (3 + k) \overline{\text{VaR}}_{60}, \text{VaR}_t \right\},$$

where $\overline{\text{VaR}}_{60}$ is the average VaR over the previous 60 trading day's and k is a multiplicative factor that depends on the number of violations in the previous 250 trading days (N_v), according to the following function,

$$k = \begin{cases} 0 & \text{if } N_v \leq 4 \\ 0.3 + 0.1(N_v - 4) & \text{if } 5 \leq N_v \leq 6 \\ 0.65 & \text{if } N_v = 7 \\ 0.65 + 0.1(N_v - 7) & \text{if } 8 \leq N_v \leq 9 \\ 1 & \text{if } N_v \geq 10(\text{red zone}). \end{cases}$$

More than 10 violations in any financial year may required to adopt "standardized" approach.

Excessive Conservatism has a negative impact on the profitability. However, ADIs are not allowed to violate more than 10 times, but any number less than 10 is permitted.

- In McAleer et al. (2010b) a risk management strategy was proposed under the Basel II Accord as being robust to the Global Financial Crisis (GFC) by selecting a Value-at-Risk (VaR) forecast that combines the forecasts of different VaR models.
- The robust forecast was based on the median of the point VaR forecasts of a set of parametric conditional volatility models.
- In this paper we provide further evidence on the suitability of the median as a GFC robust strategy, by using an additional set of Extreme Value Theory (EVT) forecasting models and by extending the sample period for comparison.
- These EVT models include Duration Peak Over Threshold, DPOT and Conditional Extreme Value Theory, CEVT.

- We investigate the performance of a variety of single and combined VaR forecasts in terms of daily capital requirements and violation penalties under the Basel II Accord, as well as other criteria, including several tests for independence of the violations.
- Our results confirm that the median remains GFCrobust even in the presence of these new nonparametric models. This is illustrated by using the S&P 500 index before, during and after the 2008-09 global financial crisis.

II - GARCH type models

We consider the GARCH and GJR with normal, t -Student and Generalized normal distribution errors. We also choose the well known RiskMetrics model.

GARCH

When the time-varying conditional variance has both autoregressive and moving average components, this leads to the generalized autoregressive conditional heteroskedasticity (GARCH) model of Bollerslev (1986).

We consider the stationary AR(1,1)-GARCH(1,1) model for daily returns y_t :

$$y_t = \varphi_1 + \varphi_2 y_{t-1} + \varepsilon_t, \quad |\varphi_2| < 1$$

for $t = 1, \dots, n$, where the shocks to returns are given by:

$$\varepsilon_t = \eta_t \sqrt{h_t}, \quad \eta_t \sim \text{iid}(0, 1)$$

$$h_t = w + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1},$$

and $w > 0$, $\alpha \geq 0$, $\beta \geq 0$ are sufficient conditions to ensure that $h_t > 0$.

GJR

In the GARCH model, the effects of positive shocks on h_t are assumed to be the same as the negative shocks. To accommodate asymmetric behaviour, Glosten, Jagannathan and Runkle (1992) proposed a model, for which GJR(1,1) is defined as follows:

$$h_t = w + (\alpha + \gamma I(\eta_{t-1}))\varepsilon_{t-1}^2 + \beta h_{t-1},$$

where $w > 0$, $\alpha \geq 0$, $\alpha + \gamma \geq 0$, $\beta \geq 0$ are sufficient conditions for $h_t > 0$, and $I(\eta_t)$ is an indicator variable defined by:

$$I(\eta_t) = \begin{cases} 1 & \varepsilon_t < 0 \\ 0 & \varepsilon_t \geq 0 \end{cases}$$

For financial data it is expected that $\gamma \geq 0$ because negative shocks have a greater impact on risk than do positive shocks of similar magnitude.

RiskMetrics

RiskMetrics (1996) developed by J.P. Morgan, estimates the conditional variance and covariances based on the exponentially weighted moving average (EWMA) method. The EWMA model calibrates the conditional variance as:

$$h_t = \lambda h_{t-1} + (1 - \lambda) \varepsilon_{t-1}^2$$

where λ is a decay parameter. RiskMetrics (1996) suggests that λ should be set at 0.94 for purposes of analysing daily data.

III - Extreme Value Theory (EVT) models

We consider two EVT models, namely Conditional EVT (CEVT) and Duration based Peaks Over Threshold (DPOT). The first is well known and widely used in the literature. The second was recently proposed by Araújo Santos and Fraga Alves (2011).

CEVT

Diebold, Schuermann and Stroughair (1998) proposed in a first step the standardization of the returns through the conditional means and variances estimated with a time-varying volatility model, and in a second step, estimation of a p quantile using EVT and the standardized returns.

McNeil and Frey (2000) combine a AR(1)-GARCH(1,1) process assuming normal innovations with the POT method from EVT. Kuester, Mittik and Paolella (2006) suggested a filter with the asymmetric skewed t distribution. We will denote this model as CEVT.

$$\widehat{\text{VaR}}_{t+1|t}^{CEVT}(p) = \hat{\mu}_{t+1|t} + \hat{\sigma}_{t+1|t} \hat{z}_p,$$

where $\hat{\mu}_{t+1|t}$ and $\hat{\sigma}_{t+1|t}$ are the estimated conditional mean and conditional standard deviation for $t + 1$, obtained with a AR(1)-GARCH(1,1) process.

Moreover, \hat{z}_p is a quantile p estimate, obtained with the POT method and the standardized residuals calculated as

$$(z_{t-n+1}, \dots, z_t) = \left(\frac{r_{t-n+1} - \hat{\mu}_{t-n+1}}{\hat{\sigma}_{t-n+1}}, \dots, \frac{r_t - \hat{\mu}_t}{\hat{\sigma}_t} \right).$$

Several studies conclude that conditional EVT is the method with better out-of-sample performance, to forecast one-day-ahead VaR: McNeil and Frey (2000), Byström (2004), Bekiros and Georgoutsos (2005), Kuester, Mittik and Paoletta (2006), Ozun, Ciffer and Yilmazer (2007), Ghorbel and Trabelsi (2008).

DPOT

The POT method is based on the excesses over u and on the Pickands-Balkema-de Haan theorem (Balkema and de Haan (1974) and Pickands(1975)). For distributions in the maximum domain of attraction of an extreme value distribution, this theorem states that when u converges to the right end point (x_F) of the distribution, the excess distribution $P[X - u|X > u]$ converges to the Generalized Pareto Distribution (GPD)

$$G_{\gamma,\sigma}(y) = \begin{cases} 1 - (1 + \gamma y/\sigma)^{-1/\gamma}, & \gamma \neq 0 \\ 1 - \exp(-y/\sigma), & \gamma = 0, \end{cases} \quad (1)$$

where $\sigma > 0$, and the support is $y \geq 0$ when $\gamma \geq 0$ and $0 \leq y \leq -\sigma/\gamma$ when $\gamma < 0$.

Smith (1987) proposed a tail estimator based on a GPD approximation to the excess distribution and inverting this estimator we get an equation to calculate the VaR forecast.

With financial time series a relation between the excesses and the durations between excesses is usually observed. Duration-based POT models (DPOT) use this dependence to improve the risk forecasts.

For estimation, these models use the durations, at time of excess i , since the preceding v excesses ($d_{i,v}$). At time t , $d_{t,v}$ denote the duration until t since the preceding v excesses. DPOT model assume the GPD for the excess Y_t above u , such that

$$Y_t \sim GPD\left(\gamma, \sigma_t = \alpha/(d_{t,v})^c\right)$$

where γ and α are parameters to be estimated. The proposed *DPOT* model implies, for $\gamma < 1$, a conditional expected value for the excess, and for $\gamma < 1/2$, a conditional variance, both dependent on $d_{t,v}$,

$$E[Y_t|\Omega_t] = \frac{\sigma_t}{1-\gamma} \quad (\gamma < 1), \quad VAR[Y_t|\Omega_t] = \frac{(\sigma_t)^2}{(1-2\gamma)} \quad (\gamma < 1/2).$$

Inverting the tail estimator based on the conditional GPD we get the equation to calculate the DPOT VaR forecast

$$\widehat{VaR}_{t+1|t}^{DPOT(v,c)}(p) = u + \frac{\hat{\alpha}}{\hat{\gamma}(d_{t,v})^c} \left(\left(\frac{n}{n_x p} \right)^{\hat{\gamma}} - 1 \right), \quad (2)$$

where $\hat{\gamma}$ and $\hat{\alpha}$ are estimators of the parameters γ and α . We choose $v = 3$ and $c \in \{2/3, 3/4\}$, since values of c close or equal to $3/4$ have been shown to exhibit the best results (Araújo Santos and Fraga Alves, 2011, TR 08/2011, CEAUL).

What is a GFC-Robust Risk Management Strategy?

Regardless of economic turbulence (tranquil or turbulent periods), a robust risk management strategy provides stable results in terms of daily capital requirements and number of violations.

IV - Comparative study - Before, During and After the GFC

We choose the S&P 500 Index returns for the periods:

Before the GFC: January 2, 2008 until August 8, 2008

During the GFC: August 11, 2008 until March 9, 2009

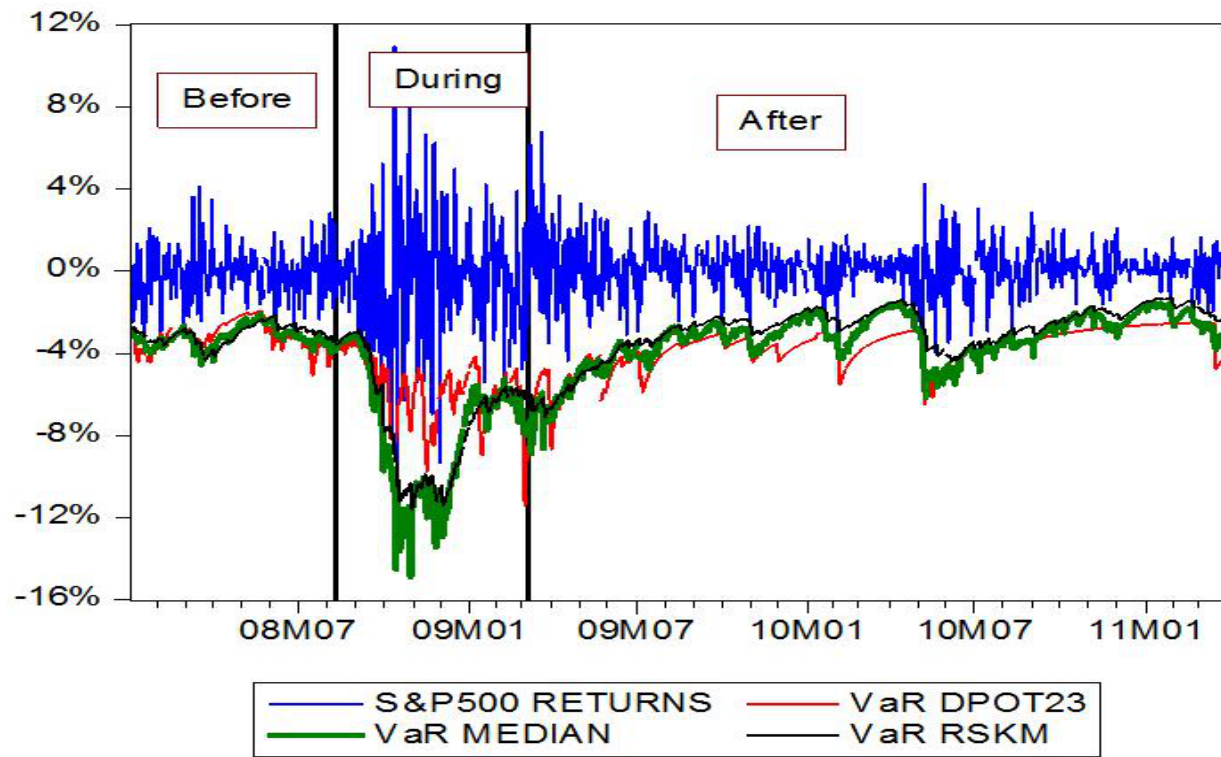
After the GFC: March 10, 2009 until March 25, 2011

We evaluate the performance in terms of capital requirements and number of violations under the the Basel II Accord.

We also evaluate the performance in terms of the unconditional coverage (UC) and independence (IND) properties. To backtesting we use the hit function

$$I_{t+1}(p) = \begin{cases} 1 & \text{if } R_{t+1} < \widehat{VaR}_{t+1|t}(p) \\ 0 & \text{if } R_{t+1} \leq \widehat{VaR}_{t+1|t}(p). \end{cases}$$

Christoffersen (1998) showed that evaluating interval forecasts can be reduced to examining whether the hit sequence satisfies the UC and IND properties. To test the unconditional coverage (UC) hypothesis we apply the Kupiec test (Kupiec, 1995). To test the independence (IND) hypothesis we apply the CAViaR independence test (Engle and Manganelli, 2004 and Berkowitz *et al.*, 2009) and a new independence test that involves a ratio between the maximum and the median of durations between violations (Araújo Santos and Fraga Alves, 2010, CSDA).



			Before			
	% of days	Average	% of	Kupiec	MM	Caviar
	In the red	DCC	violations	p value	p value	p value
RiskMetrics	0,00%	9,23	1,96%	0,29	0,52	0,28
GARCH-n	0,00%	8,52	3,92%	0,01	0,30	0,16
GARCH-t	0,00%	10,94	0,65%	0,65	-	0,36
GARCH-gnd	0,00%	9,32	2,61%	0,09	0,44	0,25
GJR-n	0,00%	8,57	3,27%	0,03	0,55	0,45
GJR-t	0,00%	10,75	0,00%	0,08	-	-
GJR-gnd	0,00%	9,33	1,96%	0,29	0,67	0,64
Cond_EVT	0,00%	10,05	1,31%	0,72	0,23	0,21
DPOT(c=2/3)	0,00%	9,16	1,96%	0,29	0,60	0,71
DPOT(c=3/4)	0,00%	9,71	0,65%	0,65	-	0,63
Infimum	0,00%	11,84	0,00%	0,08	-	-
Supremum	0,00%	7,95	3,92%	0,01	0,31	0,58
Median	0,00%	9,38	2,61%	0,09	0,45	0,44

			During			
	% of days	Average	% of	Kupiec	MM	Caviar
	In the red	DCC	violations	p value	p value	p value
RiskMetrics	0,00%	24,68	4,14%	0,00	0,45	0,02
GARCH-n	71,03%	24,44	4,83%	0,00	0,05	0,00
GARCH-t	0,00%	26,11	1,38%	0,66	0,59	0,01
GARCH-gnd	0,00%	25,72	2,07%	0,26	0,84	0,00
GJR-n	0,00%	24,16	3,45%	0,02	0,33	0,00
GJR-t	0,00%	26,06	1,38%	0,66	0,59	0,01
GJR-gnd	0,00%	25,16	2,07%	0,26	0,84	0,00
Cond_EVT	0,00%	23,64	1,38%	0,66	0,59	0,01
DPOT(c=2/3)	37,24%	19,36	5,52%	0,00	0,43	0,13
DPOT(c=3/4)	0,00%	19,73	4,83%	0,00	0,81	0,19
Infimum	0,00%	27,32	1,38%	0,66	0,59	0,30
Supremum	82,07%	18,5	8,28%	0,00	0,61	0,31
Median	0,00%	24,15	1,38%	0,66	0,59	0,35

			After			
	% of days	Average	% of	Kupiec	MM	Caviar
	In the red	DCC	violations	p value	p value	p value
RiskMetrics	21,66%	11,34	2,51%	0,00	0,04	0,03
GARCH-n	19,54%	11,55	2,90%	0,00	0,27	0,04
GARCH-t	0,00%	13,53	0,19%	0,02	-	0,18
GARCH-gnd	0,00%	11,31	1,55%	0,25	0,75	0,02
GJR-n	11,80%	11,43	3,09%	0,00	0,00	0,11
GJR-t	0,00%	12,61	0,58%	0,30	0,10	0,00
GJR-gnd	0,00%	10,47	1,55%	0,25	0,86	0,01
Cond_EVT	0,00%	11,77	0,58%	0,30	0,22	0,00
DPOT(c=2/3)	0,00%	12,99	0,39%	0,11	0,43	0,53
DPOT(c=3/4)	0,00%	12,71	0,39%	0,11	0,53	0,51
Infimum	0,00%	14,37	0,00%	0,00	-	-
Supremum	88,78%	11,11	3,87%	0,00	0,23	0,29
Median	0,00%	11	1,35%	0,44	0,14	0,02

	Number of rejections	
	UC hyp.	IND hyp.
RiskMetrics	2	3
GARCH-n	3	2
GARCH-t	1	1
GARCH-gnd	0	2
GJR-n	3	2
GJR-t	0	2
GJR-gnd	0	2
Cond_EVT	0	2
DPOT(c=2/3)	1	0
DPOT(c=3/4)	1	0
Infimum	1	0
Supremum	3	0
Median	0	1

- First we exclude from consideration the models that lead the ADIs to red in at least one period, namely: Riskmetrics, GARCH-n, GJR-n, DPOT($c=2/3$) and Supremum.
- The best model before the crisis is the GARCH-gnd, with no days in red, and the lowest average daily capital charges of 9.32%, while the temporal independence of the violations is not rejected by any of the 2 tests.
- During the crisis the best model is DPOT($c=3/4$) with no days in red and the lowest average daily capital charges of 19.73%, while independence is not rejected by any of the 2 tests, although it has a high failure rate of 4.8%.
- After the crisis, the best model is, GJR-gnd with no days in red, minimum average daily capital charges of 10.47% and independence is not rejected by 1 out of 2 tests.
- The median is respectively third, again third, and second across the three periods in terms of daily capital charges.

V - Conclusions

1. VaR models currently in use may lead to high daily capital requirements or an excessive number of violations.
2. ADIs objective is to maximize profits, so they wish to minimize their capital charges while restricting the number of violations in a given year below the maximum of 10 allowed by the Basel II Accord. From this target it follows naturally that ADIs have to choose an optimal reporting policy that may strategically under-report or over-report their forecast of VaR in order to minimize the daily capital requirement.
3. In this paper the optimal model, including a new set of non parametric models, changes from before, during and after the GFC.
4. In this paper we propose robust risk forecasts that use combinations of several conditional volatility models for forecasting VaR: eg: the median of several parametric and non parametric models.

5. The median is robust, in that it yields the lowest average daily capital charges, number of violations that do not jeopardize institutions that might use it, and more importantly, is invariant before, during and after the 2008-09 GFC.

6. The median is a model that balances daily capital charges and violation penalties in minimizing DCC.

7. Combining forecasting models is within the spirit of the Basel II Accord, although its use would require approval by the regulatory authorities, as for any forecasting model.

8. Further research is being carried out using a variety of different indexes from different countries. Tentative results confirm that the median is global financial crisis robust and clearly preferred in most cases to single models.

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